標準模型とQCDの基礎

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格子上の場の理論　夏の学校2024

於 筑波大学

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CHIBA UNIVERSITY

Opening remarks

- 北原です、よろしくお願いします。夏の学校の復活おめでとうございます。
- 専門は素粒子現象論です。2015年学位 (東京大学)、博士論文は超対称性理論における暗黒物質に対する量子補正。
- 格子QCDの専門家ではありません。QCDの専門家でもありません。
- 格子QCDが素粒子現象論の中でどのような位置付けで、どのような役割を果たしているか、最近のトピックを中心にまとめます。

強い相互作用

標準模型 Lagrangian

$$\mathcal{L}_{\mathrm{SM}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + i \sum_i \bar{\psi}_i \gamma^\mu D_\mu \psi_i - \sum_{ij} \bar{\psi}_i y_{ij} H \psi_j + \mathrm{h.c.}$$

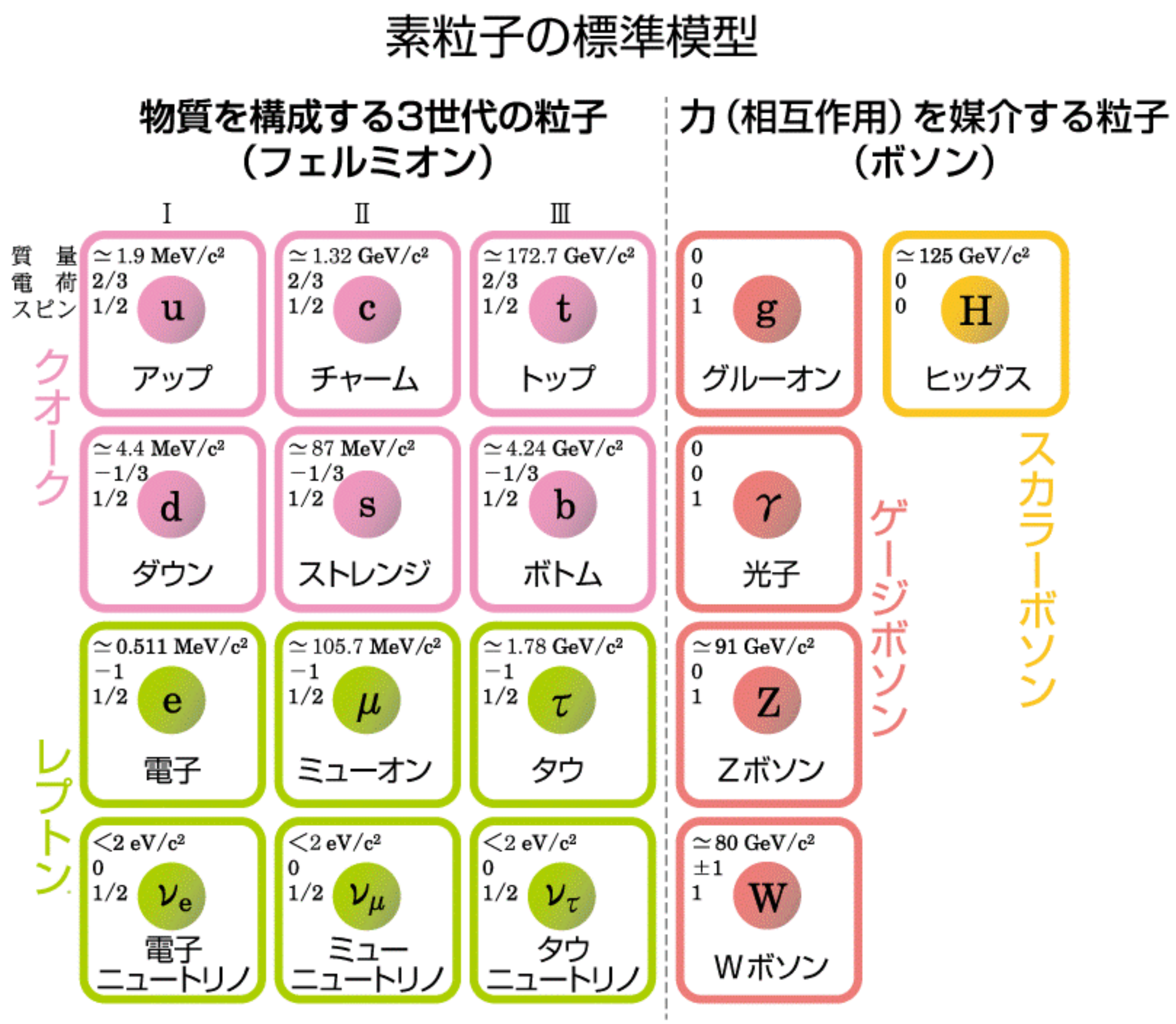
$$+ (D_\mu H)^\dagger (D^\mu H) - \mu^2 |H|^2 - \lambda |H|^4$$

強い相互作用 (QCD)

$$\mathcal{L}_{\mathrm{s}} = -\frac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} + \sum_q \bar{\psi}_{q,a} \left(i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t^A_{ab} G^A_\mu - m_q \delta_{ab} \right) \psi_{q,b}$$

with $$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_s f_{ABC} G^B_\mu G^C_\nu$$

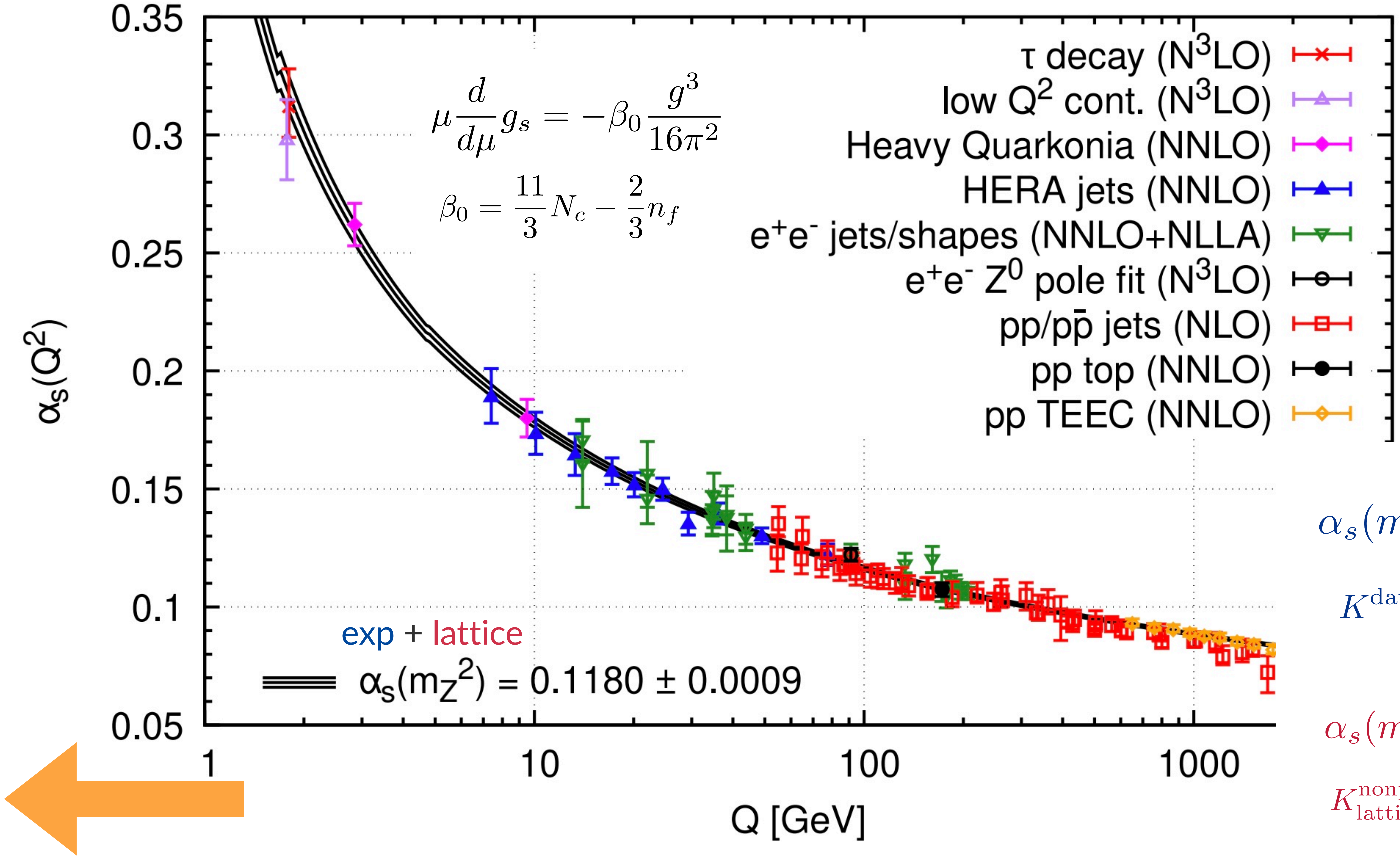
$$[t^A, t^B] = i f_{ABC} t^C \,, \qquad t^A = \lambda^A / 2$$



Running coupling $\alpha_s(\mu)$

$$\alpha_s \equiv \frac{g_s^2}{4\pi}$$

α_s	g_s
0.5	2.5
0.3	1.9
0.2	1.6
0.1	1.1



非摂動

[PDG2024]

$$\alpha_s(m_Z^2)^{\text{exp}} = 0.1175(10)$$

$$K^{\text{data}} = K^{\text{pert.}} = \sum_{i=0}^{n} c_i \alpha_s^i$$

$$\alpha_s(m_Z^2)^{\text{lattice}} = 0.1184(8)$$

$$K^{\text{nonpert.}}_{\text{lattice}} = K^{\text{pert.}} = \sum_{i=0}^{n} c_i \alpha_s^i$$

格子QCD (1/2)

- QCDはエネルギースケール $\mu \lesssim 1\text{GeV}$ では摂動計算不可能
- クォークとグルーオンの理論 ($\mu > 1\text{GeV}$) からハドロンの有効場の理論の摂動論 ($\mu < 1\text{GeV}$) に切り替わる (それでも摂動の収束性はあまり良くない)
- 非摂動領域を第一原理計算できる方法が格子QCD計算 [Wilson, 1974]
- 格子QCD計算は有限温度・有限密度にも拡張可能

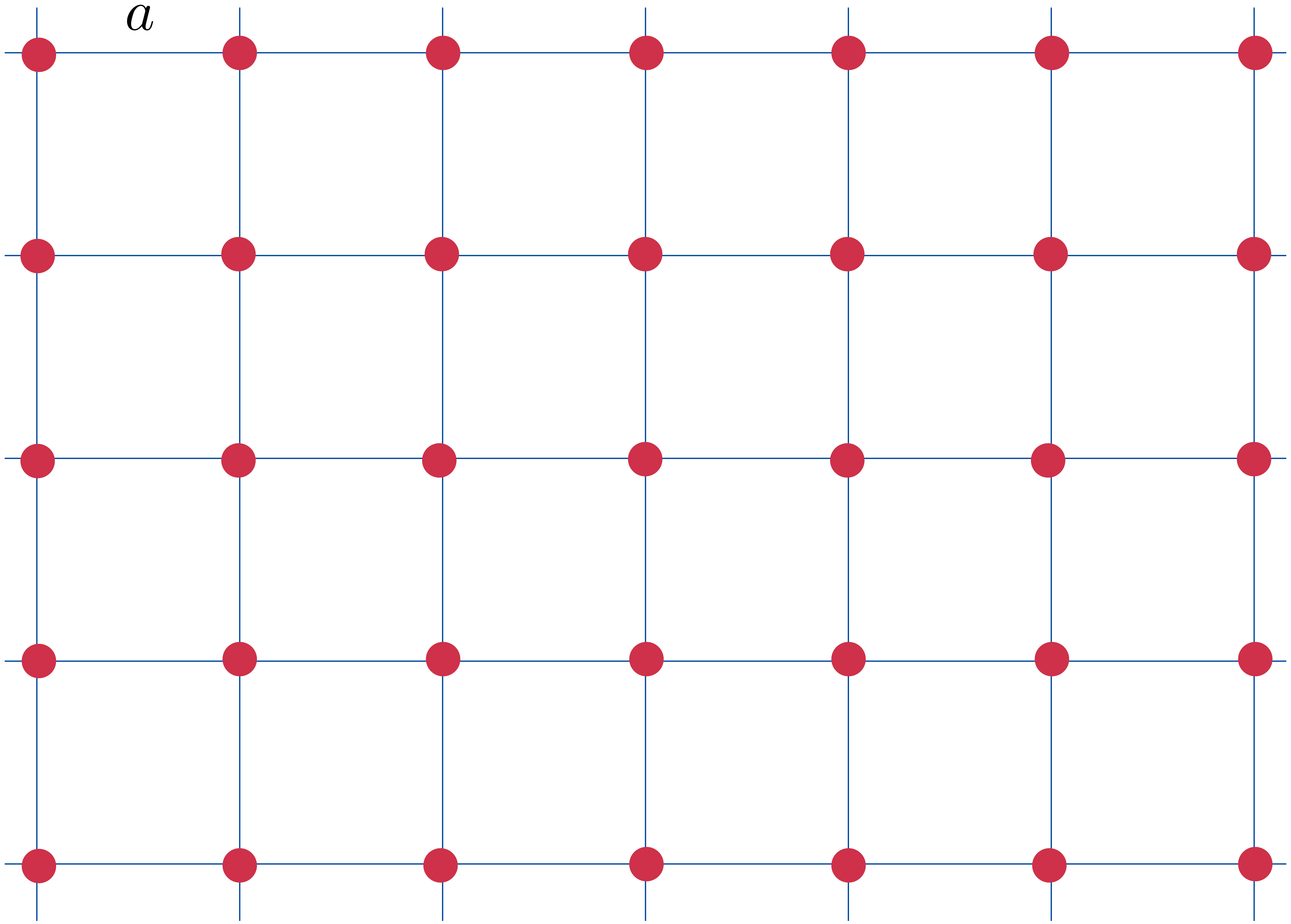
格子QCD (2/2)

格子間隔 a

クォーク場 ψ

グルーオン場 U_μ

$U_\mu = \exp(ig_s a A_\mu)$



ユークリッド
経路積分量子化

時空の離散化 =
運動量カットオフ $\dfrac{\pi}{a}$

予言 = 連続極限 $a \to 0$

ハドロンを作る =
同じ量子数を持つ
演算子の虚時間
における減衰

現象論における格子QCD

- 高エネルギー領域の物理を積分した有効ハミルトニアン

$$\mathcal{H}_{\rm eff} = \sum_i C_i(\mu) O_i(\mu)$$

Wilson係数 C　higer-dimensional 演算子 O　繰込みスケール μ

（理論家が量子効果/新物理を含め**摂動計算**）　（だいたい1GeV）

- 実験と比較可能な遷移振幅

精密測定データ ⟷
$$\mathcal{M} = \langle {\rm final}|\mathcal{H}_{\rm eff}|{\rm initial}\rangle$$
$$= \sum_i C_i(\mu)\langle {\rm final}|O_i(\mu)|{\rm initial}\rangle \equiv \sum_i C_i(\mu)\langle O_i(\mu)\rangle$$

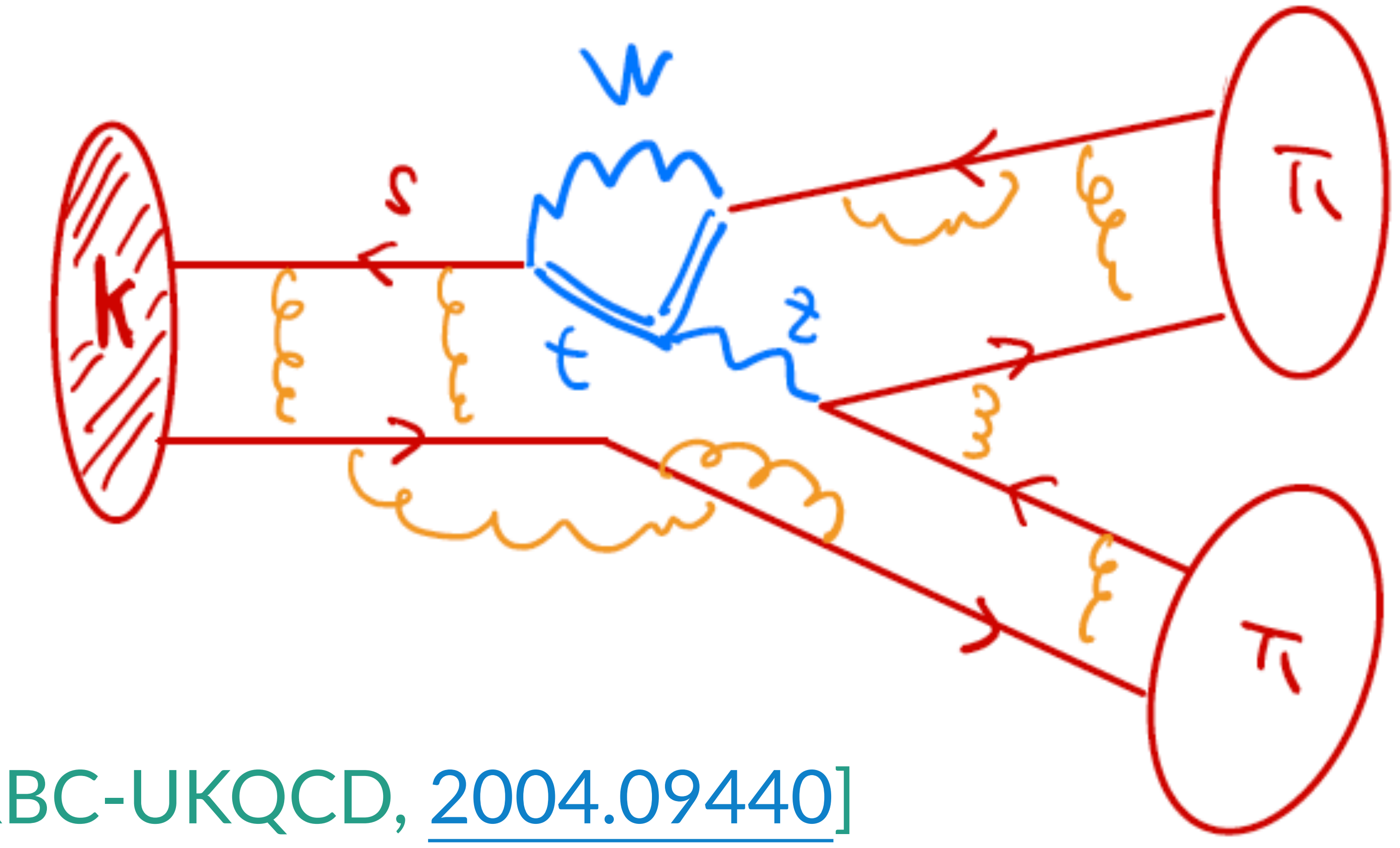
ハドロン行列要素: Hadronic Matrix Element (HME) $\langle O_i(\mu)\rangle$ ➡ 格子QCDで**非摂動計算**

成功を収めた一例

- $K_L^0 \to \pi^+\pi^-$ における Direct CP violation $\varepsilon'/\varepsilon_K$

10種類のWilson係数 (理論家) x 10種類のハドロン行列要素 (lattice)

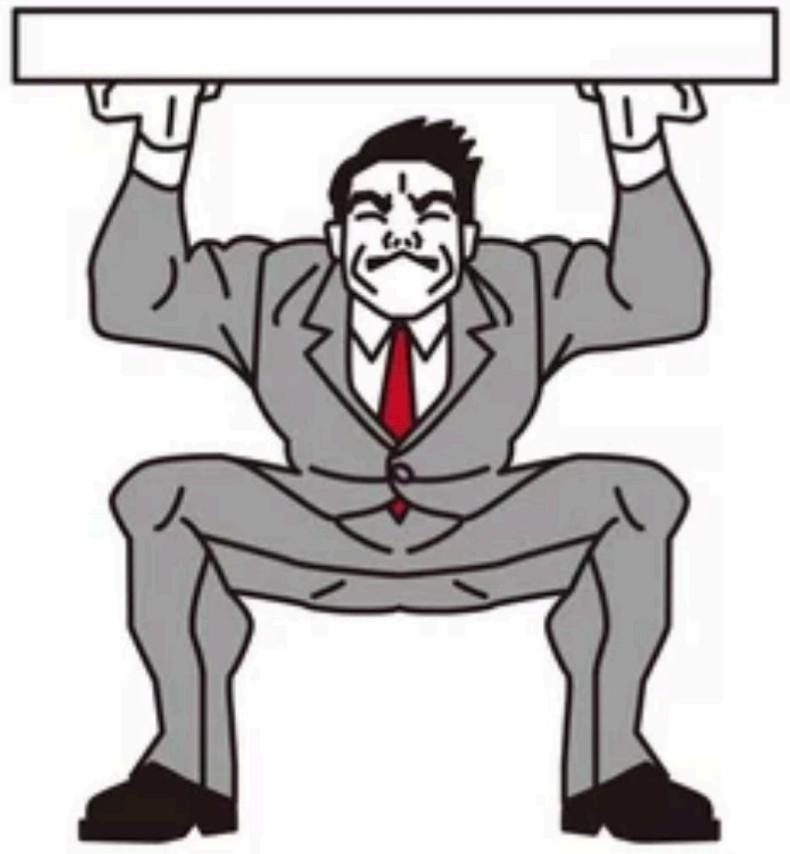
$$\langle \pi\pi \,|(\bar{s}\gamma^\mu d\gamma_5 d)(\bar{q}\gamma_\mu\gamma_5 q)|\, K\rangle$$



- 歴史的にいろいろあったが、、、最終的に一致！ [RBC-UKQCD, 2004.09440]

	Direct CP violation $\varepsilon'/\varepsilon_K$	$\pi\pi$ phase shift (I=0)	$\Delta I = 1/2$ factor
lattice QCD	$21.7(8.4) \times 10^{-4}$ $13.9(5.2) \times 10^{-4}$; improved NNLO 2005.08976	$32.3(2.1)^\circ$	$19.9(5.0)$
データ	$16.6(2.3) \times 10^{-4}$	35.9°	$22.45(6)$

Past of precision measurement physics



Present and Future

Recent progress by
the lattice QCD

created by Bing

格子QCDのホットトピック@現象論の物理

- フレーバー物理
  - K中間子, CP対称性の破れの観測量, CKM行列要素決定
  - B中間子, CKM行列要素決定
- ミューオン異常磁気双極子モーメント (g-2)
- 中性子電気双極子モーメント (EDM)

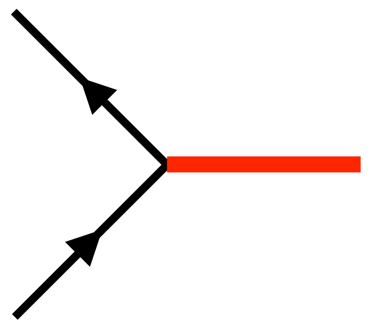
フレーバー物理 と 格子QCD

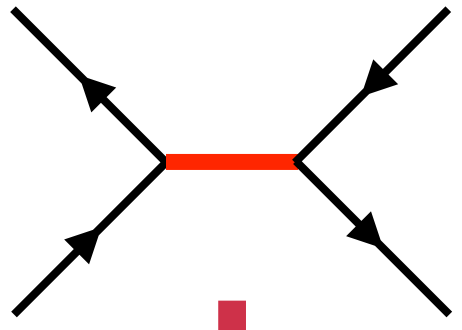
Theoretical viewpoints of Flavor physics (1/2)

- Three types of approaches to investigate new physics:
  - Fundamental physics in ideal setup and apply it to this Universe
  - Top-down: based on symmetry, breaking, and naturalness
  - **Bottom-up: based on experimental data and this Universe**
- Since quark and lepton flavor physics has a ton of experimental data, the study of flavor physics has been mainly performed by the bottom-up approach
- Ultimate goals of the flavor physics are resolving serious problems in this Universe, *e.g.*, matter-antimatter asymmetry, flavor hierarchy, neutrino mass, strong CP problem etc

Theoretical viewpoints of Flavor physics (2/2)

hunt a new physics

SM particle  New particle

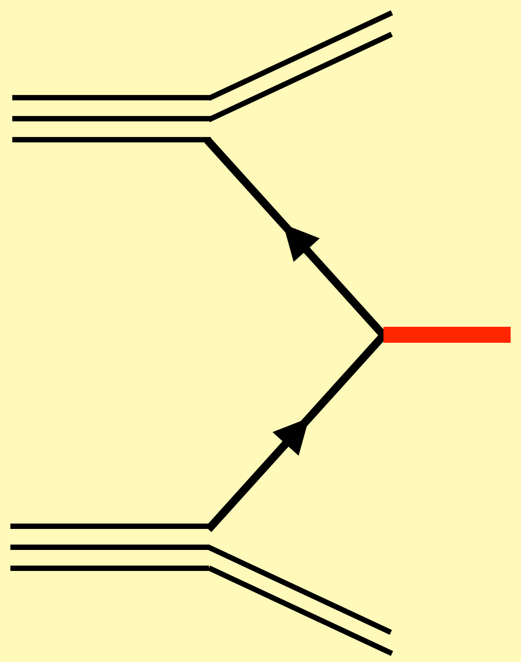


Effective field theory

Collider physics

High-energy frontier

$\sqrt{s} > m_{NP}$

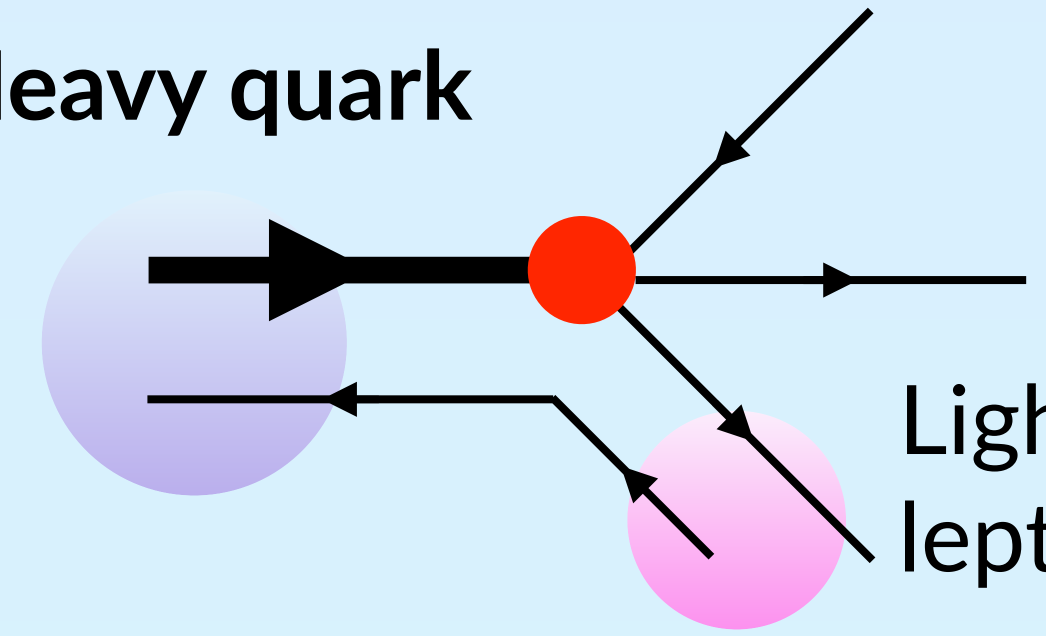


complement

Precision measurement

High-intensity frontier

Heavy quark



Light meson, lepton

Hadronization is calculable by lattice QCD

Small theoretical uncertainty → Good sensitivity to NP

Future lepton colliders (ILC, CEPC, FCC-ee, CLIC, CCC) will be capable of both frontiers

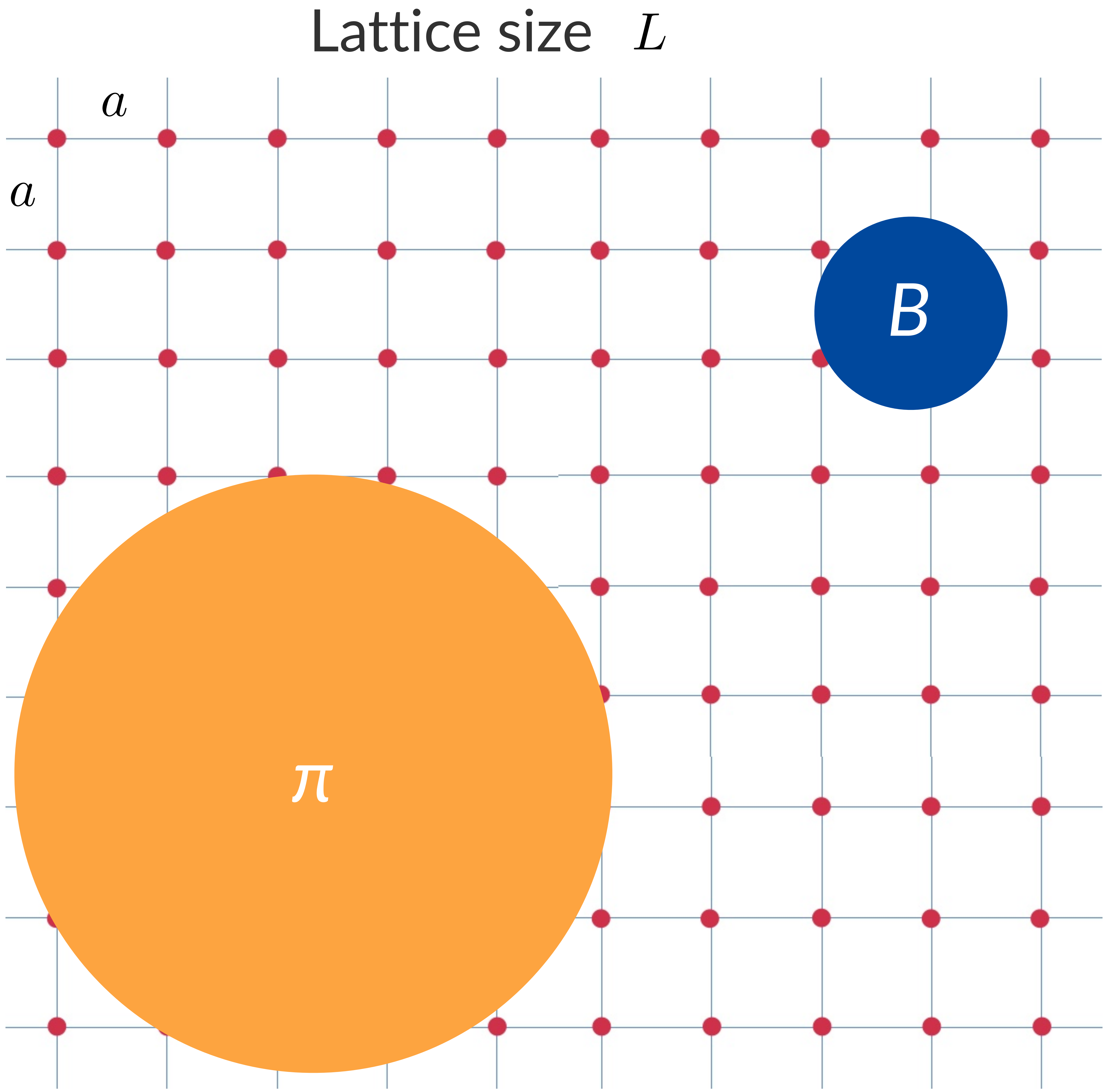
格子QCDにおけるハドロン

- マルチスケールの物理

$$\frac{1}{L} < m_\pi < m_K < m_D < m_B < \frac{1}{a}$$

- 重いハドロンの方が難しい。逆にKやD中間子は精度良く計算可能。

計算コストは $(L/a)^7 L^2$ に比例



Cabibbo-Kobayashi-Maskawa matrix (1/4)

- Cabibbo-Kobayashi-Maskawa (CKM) matrix V is introduced as misalignment of Y^u and Y^d

特異値分解: 任意の複素行列は2つのユニタリー行列を用いて実対角化可能

$$\mathcal{L} = -\bar{Q}_L Y^u U \tilde{H} - \bar{Q}_L Y^d D H + \text{h.c.}$$

$$\to -\bar{Q}_L V_{\text{CKM}}^\dagger \text{diag}(y_u, y_c, y_t) U \tilde{H} - \bar{Q}_L \text{diag}(y_d, y_s, y_b) D H + \text{h.c.}$$

with $\quad Q_L = (V_{\text{CKM}}^\dagger u_L,\, d_L)^T \qquad$ mass eigenstates u_L, d_L

then $\quad \mathcal{L}_{cc} = -g \sum_{a=1,2} \bar{Q}_L T^a \not{W}^a Q_L$

$$= -\frac{g}{\sqrt{2}} \bar{u}_L V_{\text{CKM}} \not{W}^+ d_L - \frac{g}{\sqrt{2}} \bar{d}_L \not{W}^- V_{\text{CKM}}^\dagger u_L$$

A tree-level flavor-changing charged current appears

Cabibbo-Kobayashi-Maskawa matrix (2/4)

- Within the SM, the CKM matrix has to be the 3×3 unitary matrix
  - If the unitarity violation is observed, it is clear signal of the new physics

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

If SM is correct : unitarity is satisfied

$$V^\dagger V = V V^\dagger = 1$$

$$V^\dagger V \neq V V^\dagger \neq 1$$

New physics signal

Cabibbo-Kobayashi-Maskawa matrix (3/4)

- In the SM, the CKM matrix produces single CP-violating phase [Kobayashi, Maskawa, ‘73]

N-generation CKM

$$\underbrace{2N^2 - N^2}_{\text{unitarity}} - \underbrace{\frac{N(N-1)}{2}}_{\text{rotation}} - \underbrace{(2N-1)}_{\text{quark redefinition}} = \frac{1}{2}(N-1)(N-2)$$

# of CPV phase

- The complex 9 components can be parametrized only 4 parameters in the SM

9 flavor-changing processes

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

overconstrain

SM prediction = 4 parameters

3 rotation angles
+ 1 CP-violating phase

Cabibbo-Kobayashi-Maskawa matrix (4/4)

- The precision determinations of the CKM/PMNS elements are crucial to understanding the origin of quark/lepton flavor
- Each component can be measured independently without imposing the unitarity
- One can test the CKM unitarity conditions from data:

K meson decays

B meson decays

β decays

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

D meson decays

K and B mesons mixing

$VV^\dagger = \mathbb{I}_3$ or $\neq \mathbb{I}_3$?

SM NP?

CKM成分測定の例

- $K \to \pi \ell \nu$ の崩壊チャネル ($K^+, K^0_L, K^0_S \times e, \mu$ の6種) のデータと理論の比較

理論式

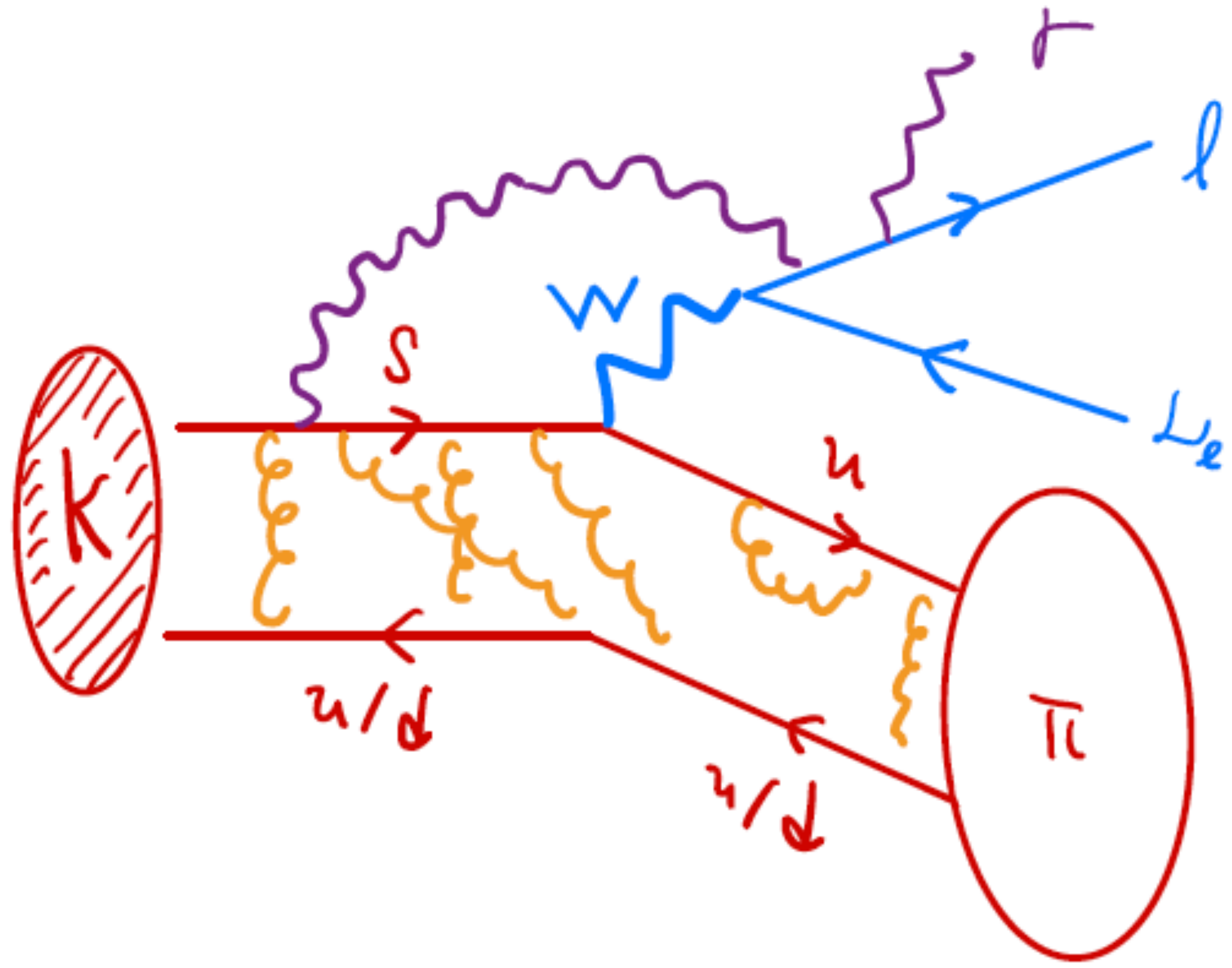
量子効果　　　　位相空間　量子効果

$$\Gamma(K \to \pi\ell\nu) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\rm EW} \, |V_{us}|^2 \left| f_+^{K^0\pi^-}(0) \right|^2 I_{K\ell} \left(1 + \delta_{\rm EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)^2$$

ハドロン行列要素

格子QCD $q \equiv p - p'$

$$\langle \pi(p') | \bar{s}\gamma_\mu u | K(p) \rangle = \left(p_\mu + p'_\mu + \frac{m_K^2 - m_\pi^2}{q^2} q_\mu \right) f_+(q^2) + \frac{m_K^2 - m_\pi^2}{q^2} q_\mu f_0(q^2)$$



データと $\Gamma(K \to \pi\ell\nu)$ の比較 ➡ $|V_{us}|$ の決定

Current data without unitarity

- PDG averages without imposing the unitarity condition [PDG2024]

CAA

inc. vs exc. tentions

scale factor 2.5 (us)

scale factor 1.4 (ub)

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97367(32) & 0.22431(85) & 0.00382(20) \\ 0.221(4) & 0.975(6) & 0.0411(12) \\ 0.0086(2) & 0.0415(9) & 1.010(27) \end{pmatrix}$$

$$\frac{|\delta V_{\mathrm{CKM}}|}{|V_{\mathrm{CKM}}|} = \begin{pmatrix} 0.033 & 0.38 & 5.2 \\ 1.8 & 0.62 & 2.9 \\ 2.3 & 2.2 & 2.7 \end{pmatrix} \%$$

scale factor 3.0 (cb)

Even if one includes the scale factor, the 1st-row unitarity violation is shown at 2.3σ [PDG2024]

CKM unitarity triangle (1/2)

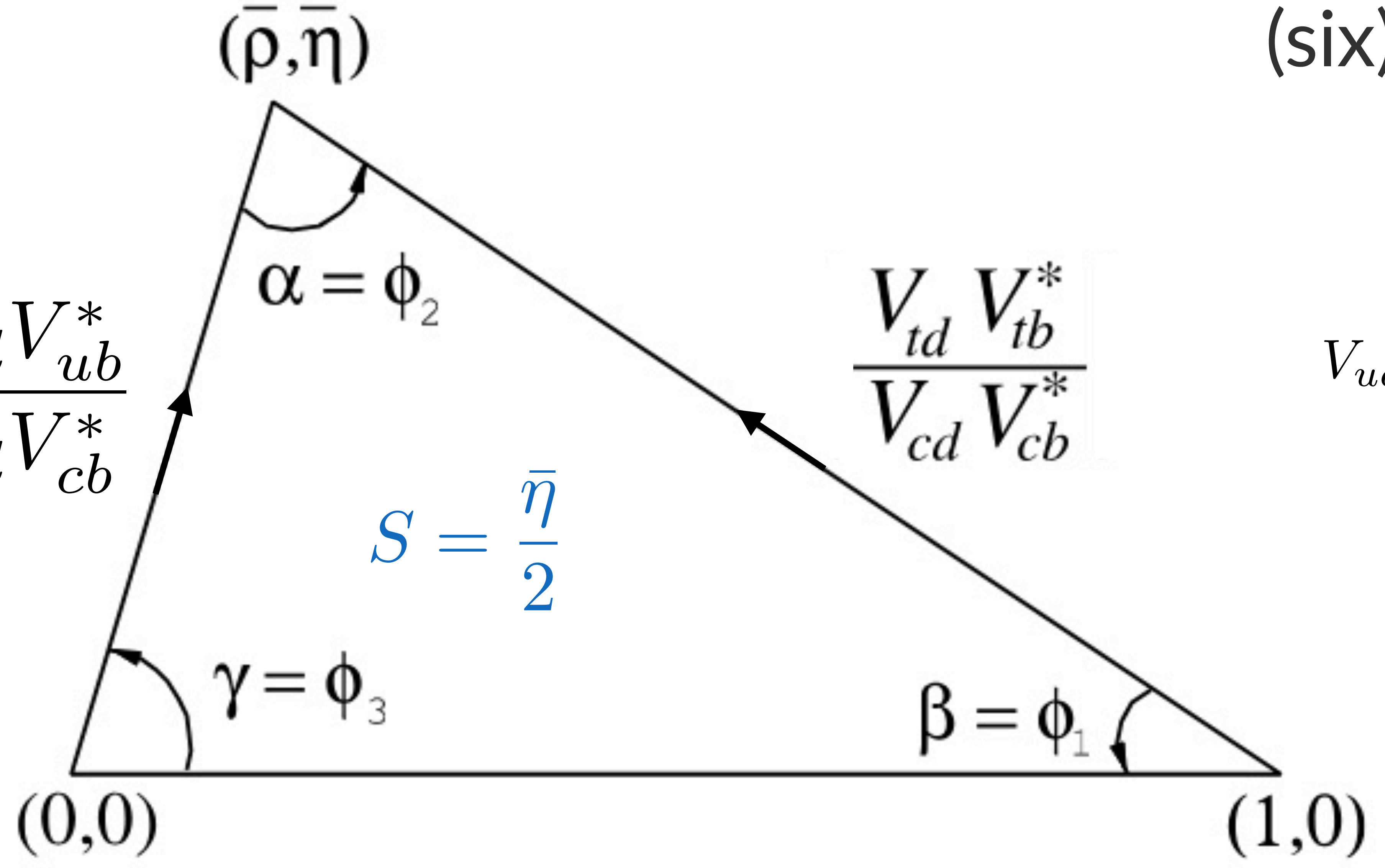
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Unitarity condition

$$V^\dagger V = \mathbb{I}_3$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

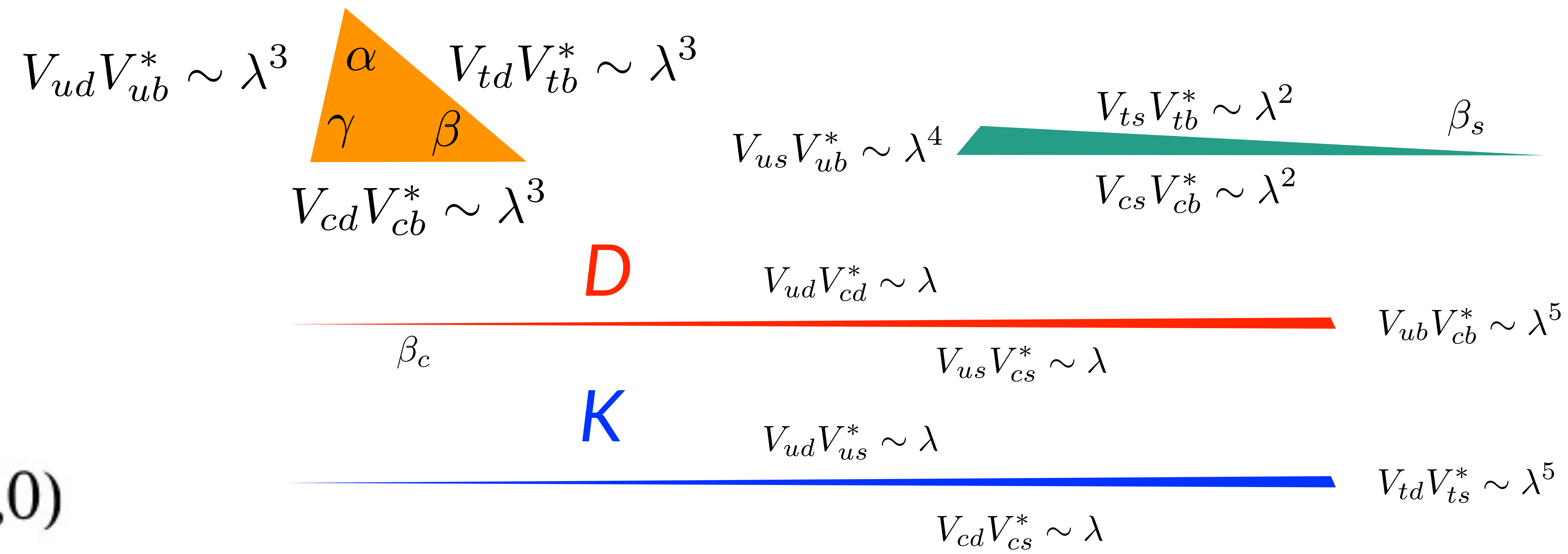
$$\bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$



(six) triangles can be drawn on a complex plane

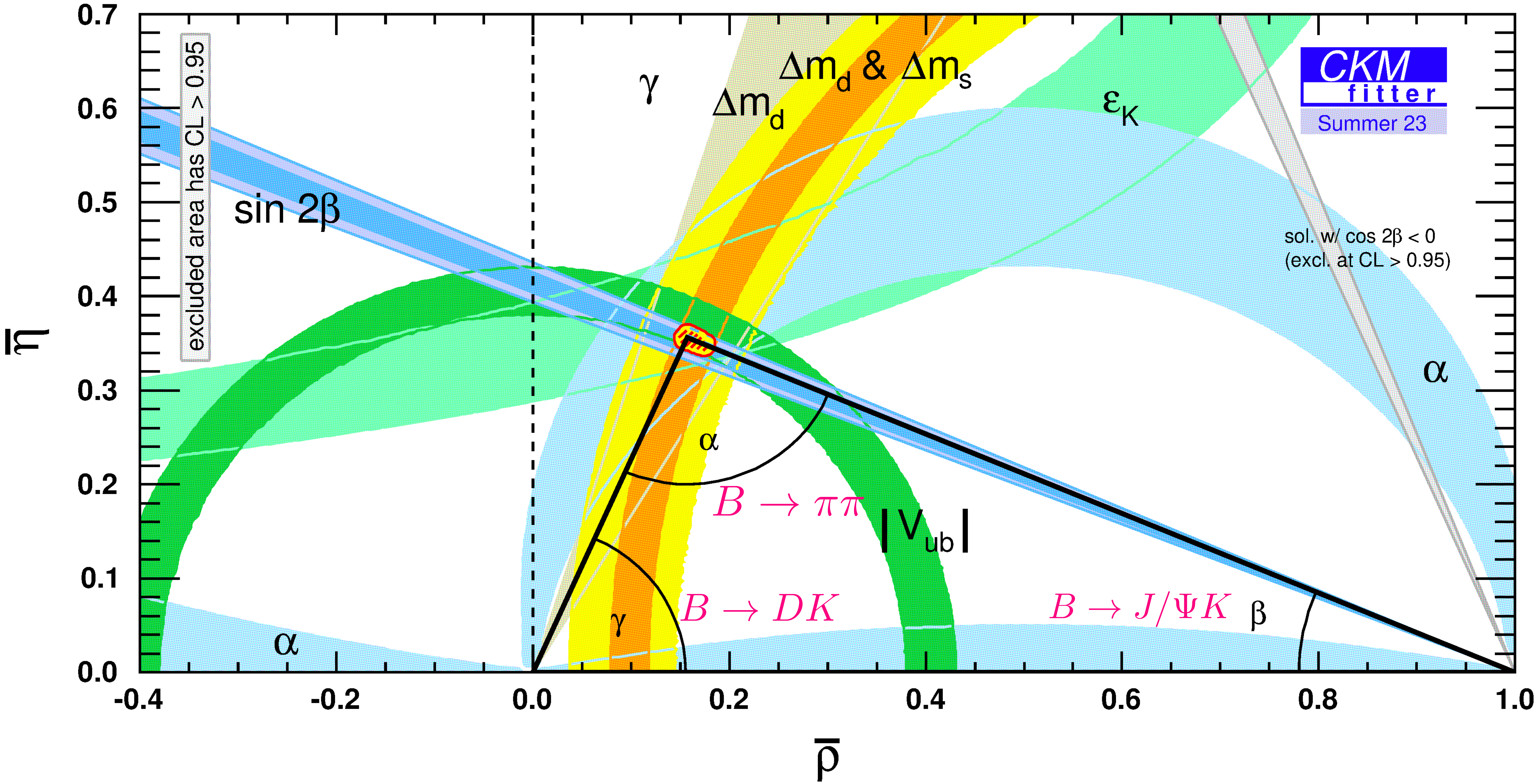
B triangle

Bs triangle



CKM unitarity triangle (2/2)

[CKM fitter 2023]



CPV

fit with unitarity
[CKM fitter 2023]

$$\lambda = 0.22498^{+23}_{-21}$$
$$A = 0.8215^{+47}_{-82}$$
$$\bar\rho = 0.1562^{+112}_{-40}$$
$$\bar\eta = 0.3551^{+51}_{-57}$$

without unitarity
[PDG2024]

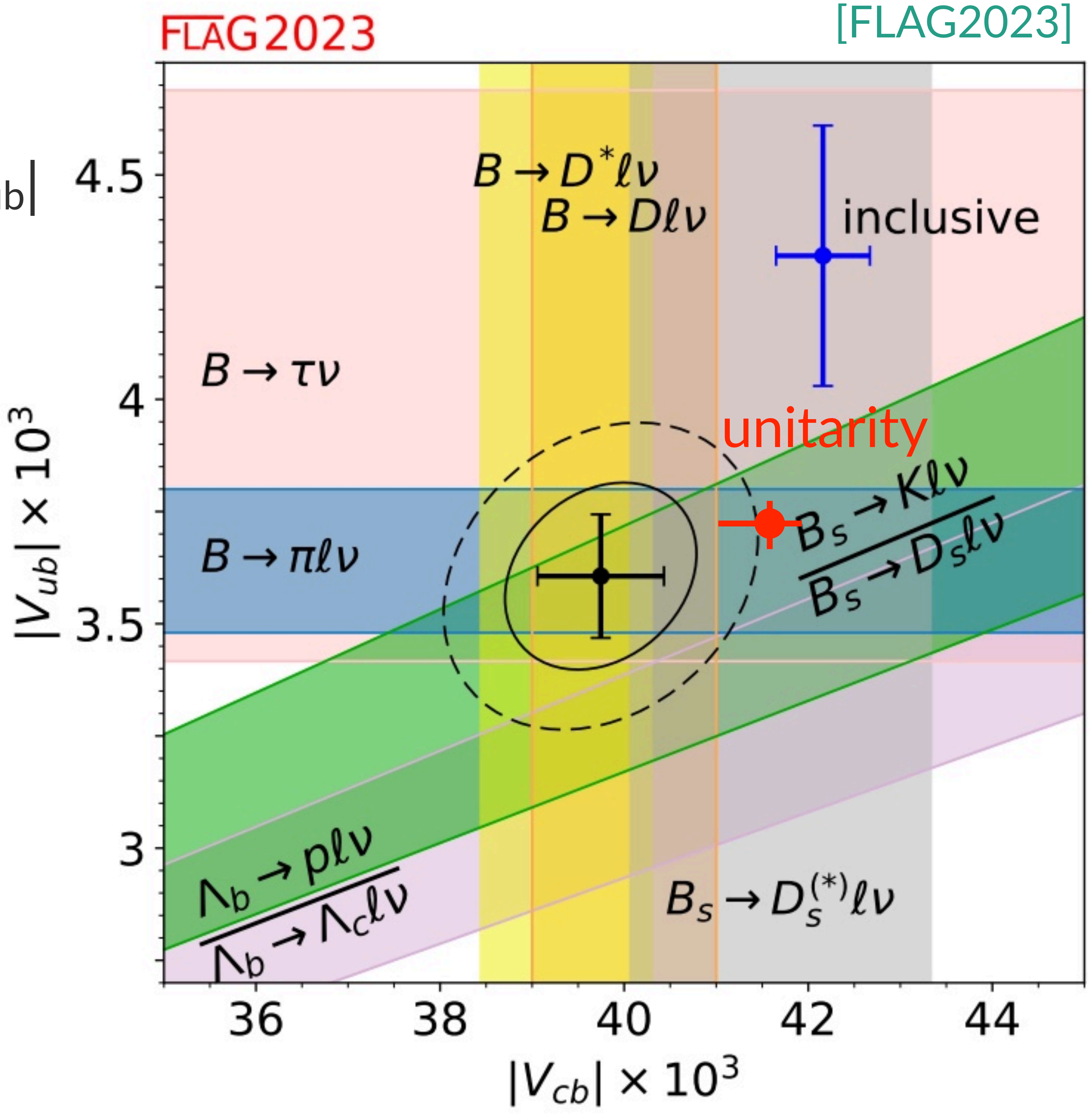
$$\alpha + \beta + \gamma = 172(5)^\circ$$

$|V_{cb}|$ – $|V_{ub}|$ plane

- Global fit of the exclusive decays (black contour) provides lower values of $|V_{cb}|$ and $|V_{ub}|$
- Inclusive one gives higher values
- Long standing ~3σ tension between the exclusive and inclusive determinations
- Unitarity condition gives a different point [CKMfitter2023]
- Belle will give a lager V_{cb}^{excl} input ($B \to D^*$)

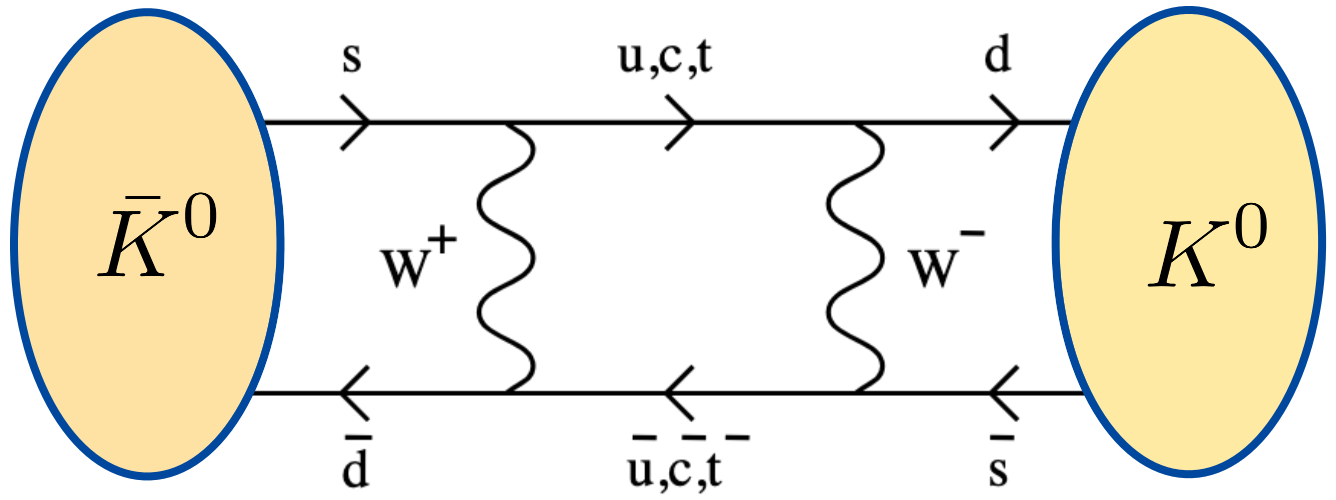
$$|V_{cb}|^{\mathrm{BGL}} = 41.0(7) \times 10^{-3}$$ [Belle, 2310.20286]

[FLAG2023]



Kaon-mixing CPV ε_K vs $|V_{cb}|$

- The indirect *CP* violation ε_K in kaon decays ($K_L \to \pi\pi$) is very sensitive to $|V_{cb}|$



$$\varepsilon_K \simeq -\mathrm{Im}(V_{ts}^* V_{td})\mathrm{Re}(V_{ts}^* V_{td})\eta_{tt} S_{\mathrm{box}}(x_t) \quad \longleftarrow \text{no lattice uncertainty}$$

$$\simeq A^4 \lambda^{10} \bar{\eta}(1-\bar{\rho})\eta_{tt} S_{\mathrm{box}}(x_t) \quad \longleftarrow \text{unitarity condition}$$

$$= |V_{cb}|^4 \lambda^2 \bar{\eta}(1-\bar{\rho})\eta_{tt} S_{\mathrm{box}}(x_t)$$

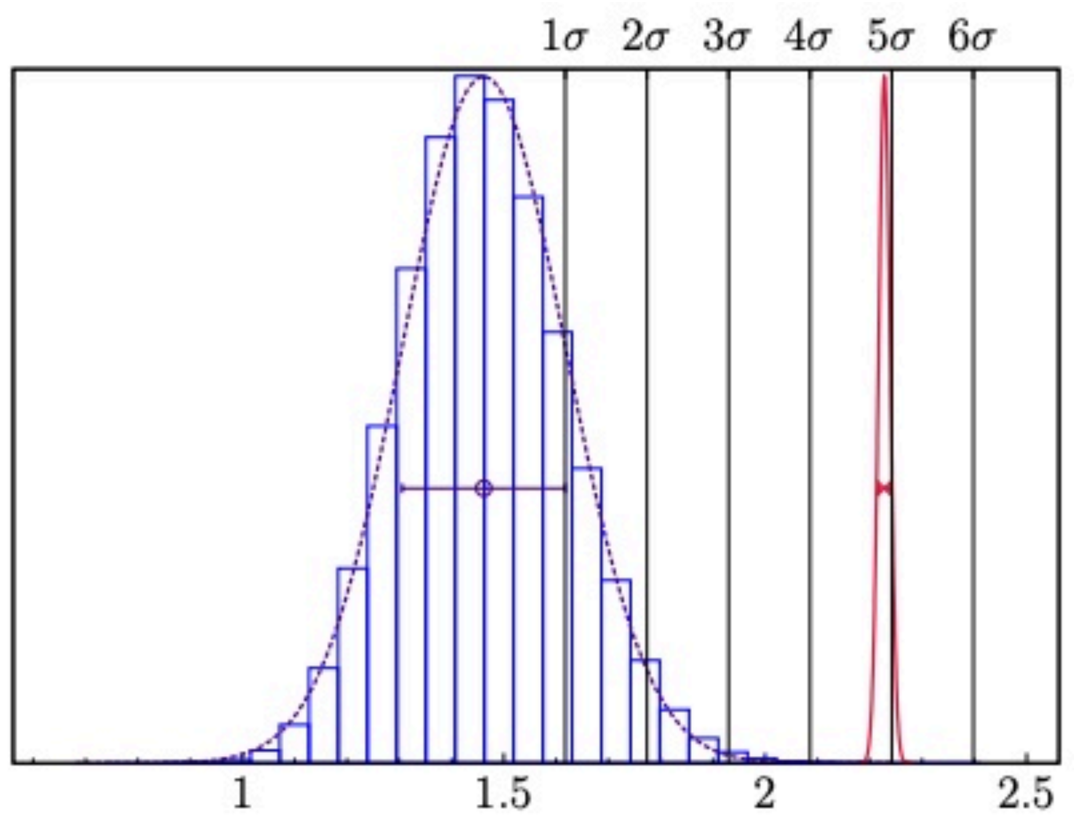
Proportional to $|V_{cb}|^4$

Exclusive Vcb gives a lower value of the SM prediction of $|\varepsilon_K|$, 4.9σ deviation from data

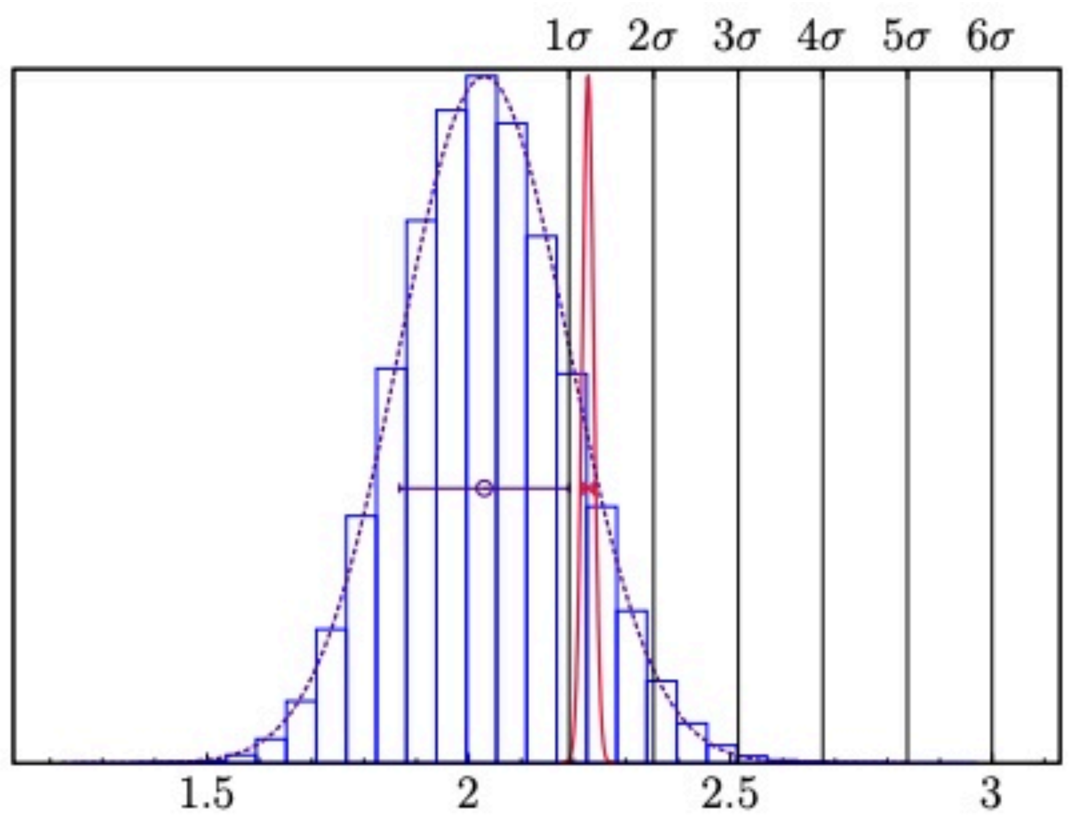
[SWME collaboration, 2312.02986]

Kaon physics + unitarity obviously prefer inclusive V_{cb} value

$|\varepsilon_K|^{\mathrm{exp}}$



exclusive V_{cb} prediction



inclusive V_{cb} prediction

1st-row Unitarity test in CKM matrix

- Thanks to the improvement of lattice QCD, the 1st-row unitarity test is currently excited

$$\begin{pmatrix} \boxed{V_{ud} \quad V_{us} \quad V_{ub}} \\ V_{cd} \quad V_{cs} \quad V_{cb} \\ V_{td} \quad V_{ts} \quad V_{tb} \end{pmatrix}$$

Unitarity condition

$$VV^\dagger = \mathbb{I}_3$$

$\Rightarrow$

1st-row unitarity condition

$$|V_{ud}|^2 + |V_{us}|^2 + \cancelto{10^{-5}}{|V_{ub}|^2} = 1$$

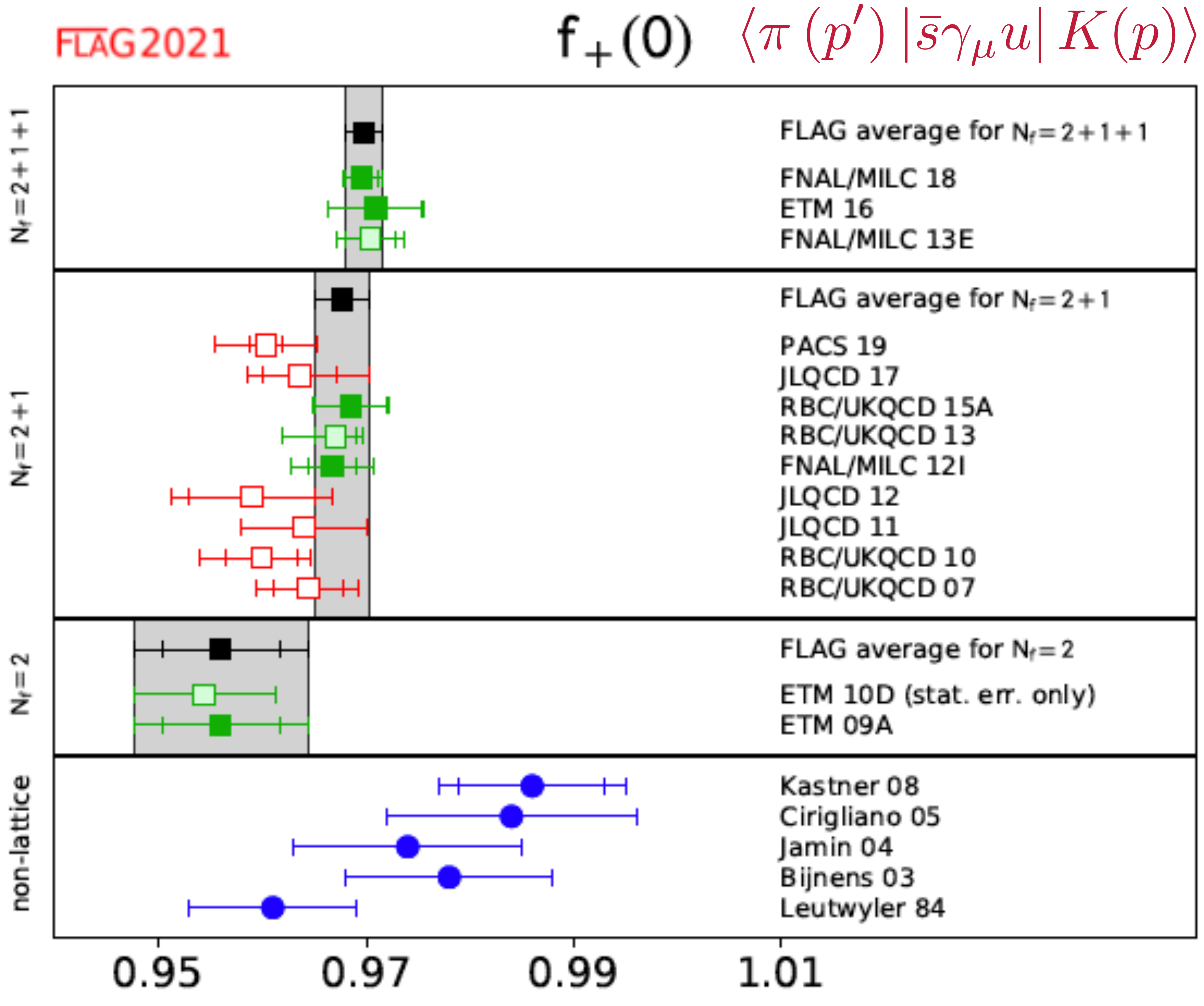
Sum of the absolute values must become **exact 1 within the SM**

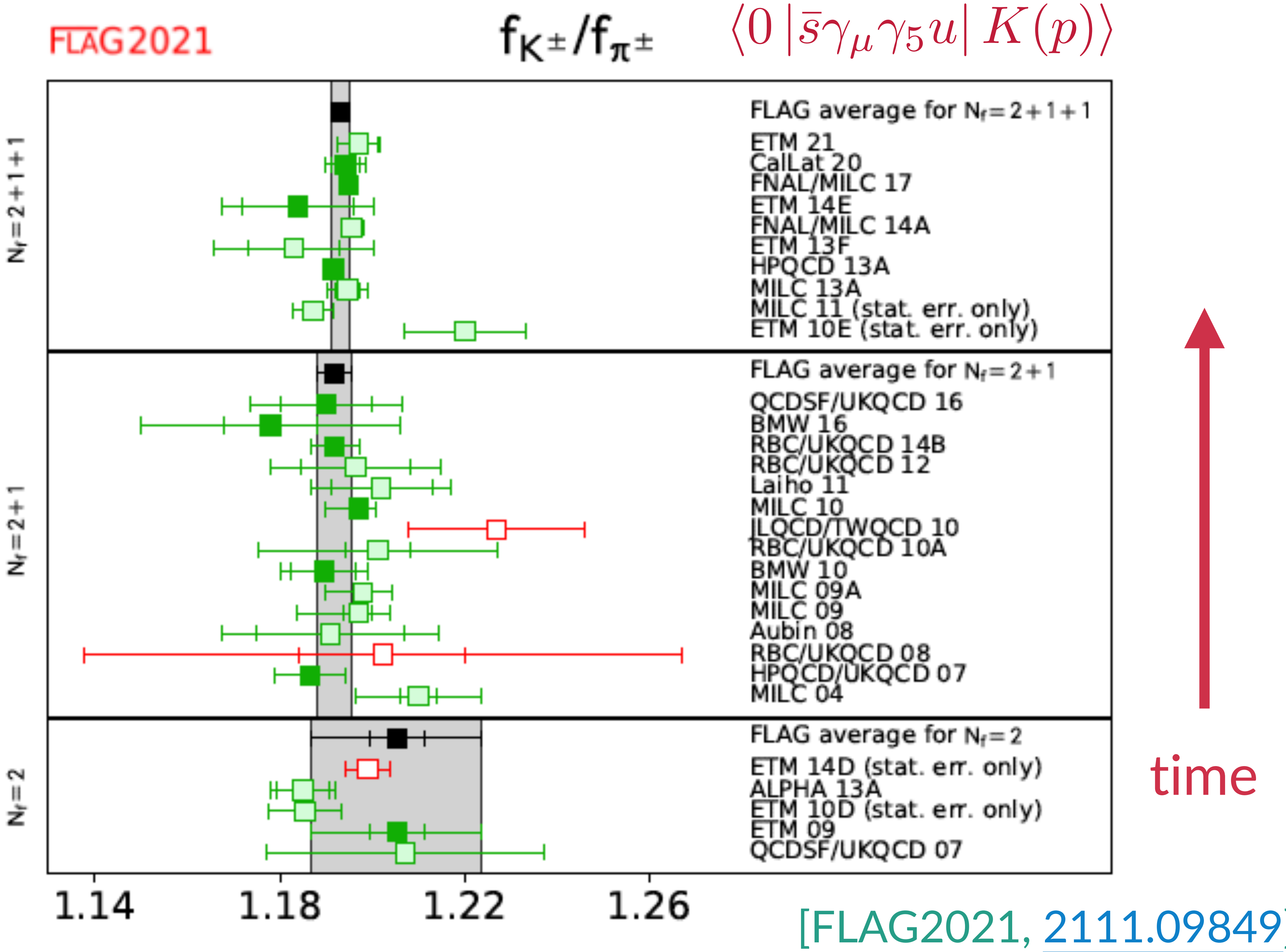
$$VV^\dagger = \mathbb{I}_3 \text{ or } \neq \mathbb{I}_3 ?$$

SM NP?

Lattice improvement

- Leading uncertainties from kaon form factors have been improved significantly

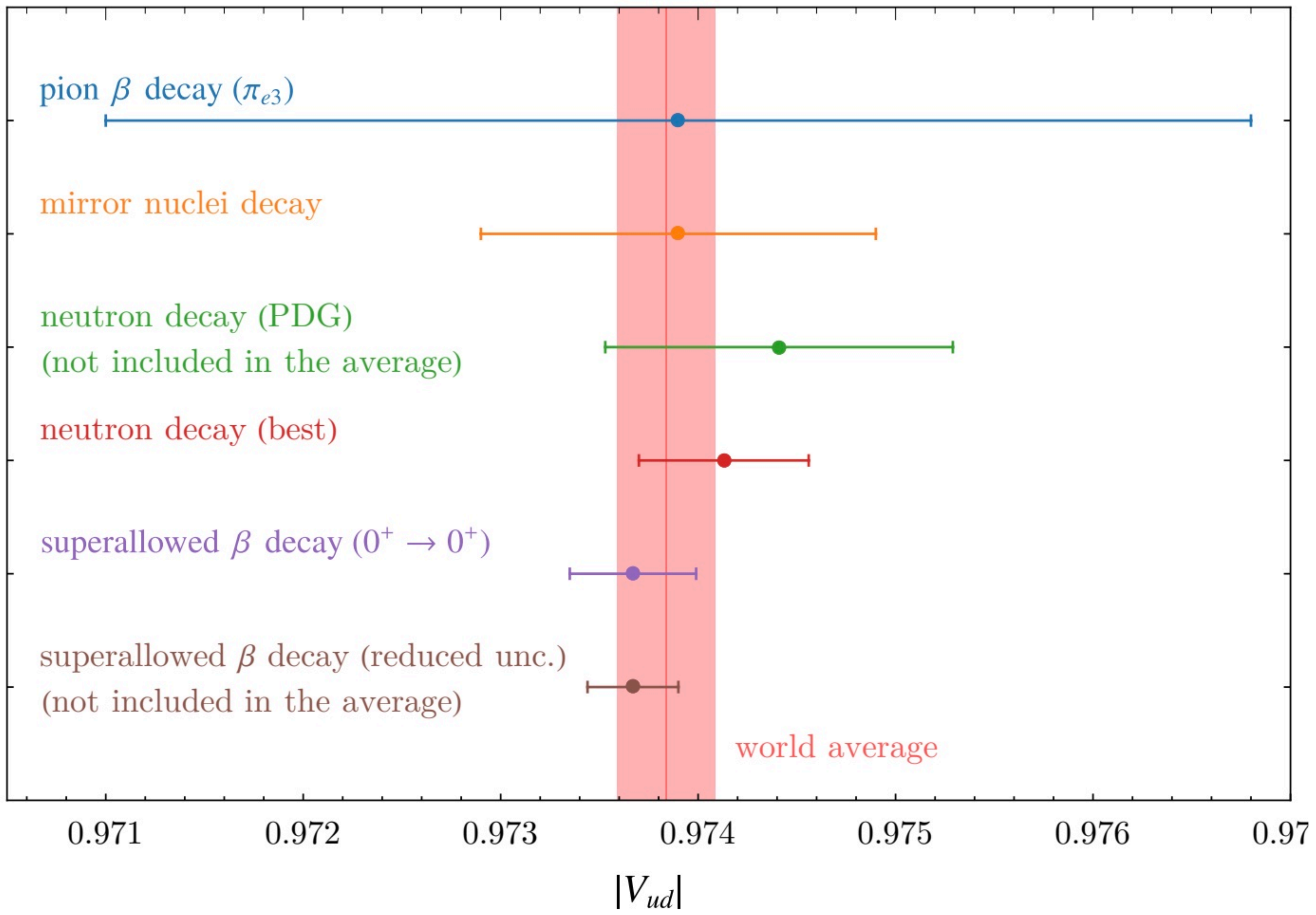


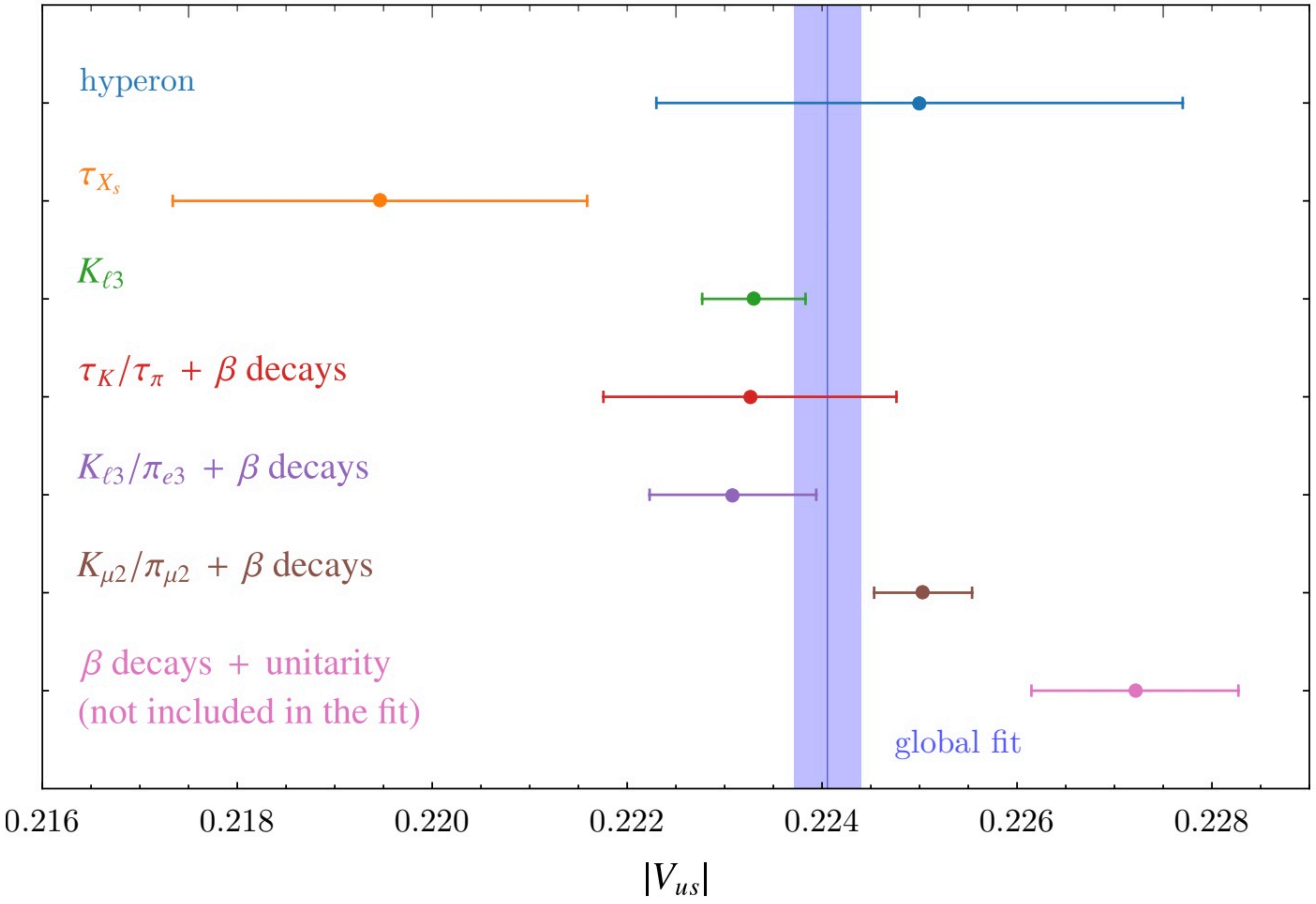


[FLAG2021, 2111.09849]

$|V_{ud}|$ and $|V_{us}|$ determinations

[Crivellin, Kirk, TK, Mescia, 2212.06862]





* new lattice result for super-allowed β decays is not included [Ma, et al., 2308.16755] (reduce CAA tension 0.5σ level)

One can see several tensions in $|V_{us}|$ determinations

Global fit of $|V_{ud}|$ and $|V_{us}|$

[Crivellin, Kirk, TK, Mescia, 2212.06862]

$K_{\ell 3}$

$K^0_{L,S} \to \pi^+ \ell \bar{\nu}$

$K^- \to \pi^0 \ell \bar{\nu}$

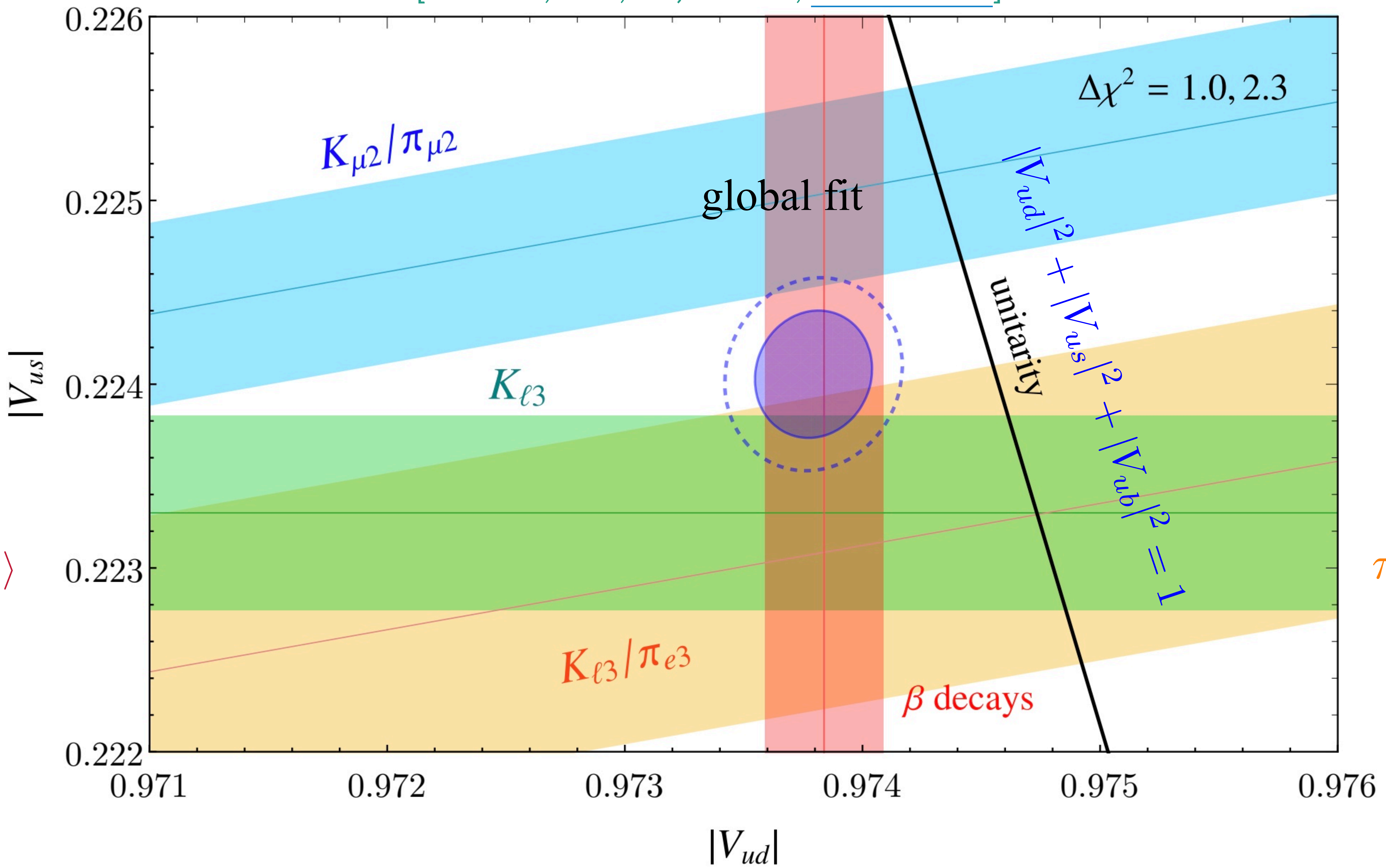
$(\ell = e,\ \mu)$

Error budgets:

LO: data, FFs

$\langle \pi(p') | \bar{s}\gamma_\mu u | K(p) \rangle$

NLO: Isospin breaking correction



$K_{\mu 2}/\pi_{\mu 2}$

$$\frac{K^- \to \mu \bar{\nu}}{\pi^- \to \mu \bar{\nu}}$$

Error budgets:

LO: FFs

$\langle 0 | \bar{s}\gamma_\mu \gamma_5 u | K(p) \rangle$

NLO: data, radiative correction

$\pi_{e3} : \pi^+ \to \pi^0 e^+ \nu$

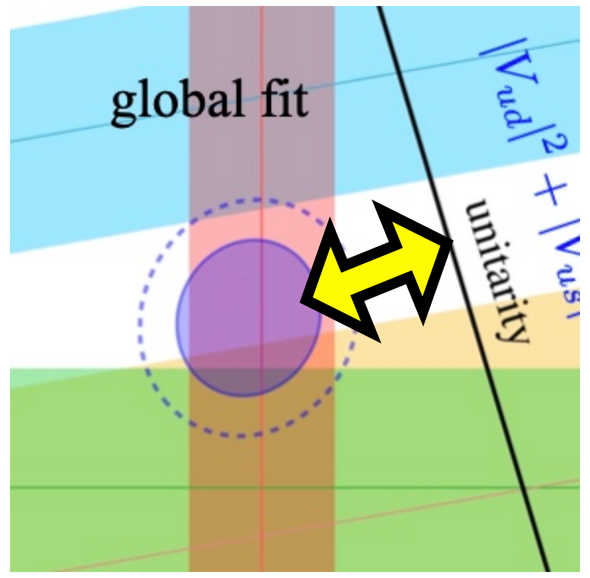
Error budgets:

LO: data

Uncertainty from $|V_{ub}|$ is negligible

Significance of Cabibbo-Angle Anomaly (CAA)

- Global fit (including with correlations) [Crivellin, Kirk, TK, Mescia, 2212.06862]



our result $\begin{cases} |V_{ud}|_{\text{global}} = 0.973\,79(25)\,, & \text{w/ bottle UCN best} \\ |V_{us}|_{\text{global}} = 0.224\,05(35)\,, & \rho(V_{ud}, V_{us}) = 0.09 \end{cases}$

the single most precise data $\begin{cases} \tau_n^{\text{bottle}} = 877.75(36)\text{sec} & |V_{ud}|_n^{\text{bottle}} = 0.974\,13(43) \\ \tau_n^{\text{beam}} = 887.7(2.2)\text{sec} & |V_{ud}|_n^{\text{beam}} = 0.968\,66(131) \end{cases}$

Long-standing 4σ inconsistency (neutron lifetime anomaly)

neutron-lifetime data dependence (bottle vs beam)

test of unitarity

$$\Delta_{\text{CKM}}^{\text{global}} \equiv |V_{ud}|^2_{\text{global}} + |V_{us}|^2_{\text{global}} + |V_{ub}|^2 - 1 = \begin{cases} -1.51(53) \times 10^{-3} & (\text{w/ bottle UCN best})\,, \\ -2.34(62) \times 10^{-3} & (\text{w/ in-beam best})\,, \end{cases}$$

-2.8σ **(UCN)** and -3.8σ level (in-beam) deviations from SM [TK, Tobioka, 2308.13003]

SMEFT

- In general new physics scenario, if the NP scale is much higher than the EW scale, one can consider the dimension-six Standard Model Effective Field Theory (SMEFT), where $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry is conserved below cutoff scale Λ

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i C_i Q_i$$

[Grzadkowski, et al., 1008.4884]

independent dimension-six operator contributing to CKM non-unitarity

$$Q_{Hq}^{(1)ij} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_i \gamma^\mu P_L q_j)\,, \qquad Q_{Hq}^{(3)ij} = (H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{q}_i \tau^I \gamma^\mu P_L q_j)\,,$$

$$Q_{Hu}^{ij} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_i \gamma^\mu P_R u_j)\,, \qquad Q_{Hd}^{ij} = (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_i \gamma^\mu P_R d_j)\,,$$

$$Q_{Hud}^{ij} = i(\tilde{H}^\dagger D_\mu H)(\bar{u}_i \gamma^\mu P_R d_j)\,.$$

Modified W and Z couplings

- After the spontaneous electroweak symmetry breaking $\langle H^0 \rangle = v/\sqrt{2}$ with $v = 246$ GeV,
  W and *Z* quark currents are modified

SM terms

Observed V_{CKM} matrix (non-unitarity)

$$
\begin{aligned}
\mathcal{L}_{W,Z} = & -\frac{g_2}{\sqrt{2}} W_\mu^+ \bar{u}_i \gamma^\mu \left(\left[V \cdot \left(1 + v^2 C_{Hq}^{(3)} \right) \right]_{ij} P_L + \frac{v^2}{2} \left[C_{Hud} \right]_{ij} P_R \right) d_j + \text{ h.c.} \\
& - \frac{g_2}{6 c_W} Z_\mu \bar{u}_i \gamma^\mu \left(\left[\left(3 - 4 s_W^2 \right) + 3 v^2 V \cdot \left\{ C_{Hq}^{(3)} - C_{Hq}^{(1)} \right\} \cdot V^\dagger \right]_{ij} P_L - \left[4 s_W^2 + 3 v^2 C_{Hu} \right]_{ij} P_R \right) u_j \\
& - \frac{g_2}{6 c_W} Z_\mu \bar{d}_i \gamma^\mu \left(\left[\left(2 s_W^2 - 3 \right) + 3 v^2 \left\{ C_{Hq}^{(3)} + C_{Hq}^{(1)} \right\} \right]_{ij} P_L + \left[2 s_W^2 + 3 v^2 C_{Hd} \right]_{ij} P_R \right) d_j
\end{aligned}
$$

- Non-unitary V_{CKM} provides non-trivial effects to *Z* currents including FCNCs

SMEFT fitting for CAA

[Grossman, Passemar, Schacht, 1911.07821]
[Crivellin, Kirk, TK, Mescia, 2212.06862]

- SMEFT global fit implies right-handed W-u-d and W-u-s currents C_{Hud} are preferred

[unit of $(10^{-3}v^{-2})$]

no contribution to $D-\bar{D}$ mixing

EFT Scenario	Best fit point	$-\Delta\chi^2$	Pull
$\left[C_{Hq}^{(3)}\right]_{11}$	-0.50	3.3	$1.8\,\sigma$
$\left[C_{Hq}^{(3)}\right]_{11} = \left[C_{Hq}^{(3)}\right]_{22}$	-0.27	1.1	$1.1\,\sigma$
$\left[C_{Hq}^{(3)}\right]_{11} = \left[C_{Hq}^{(1)}\right]_{11}$	-0.55	3.7	$1.9\,\sigma$
$[C_{Hud}]_{11}$	-1.0	3.1	$1.8\,\sigma$
$[C_{Hud}]_{12}$	-2.0	7.4	$2.7\,\sigma$
$([C_{Hud}]_{11}, [C_{Hud}]_{12})$	$(-1.4, -2.1)$	13	$3.2\,\sigma$
$\left(\left[C_{Hq}^{(3)}\right]_{11}, [C_{Hud}]_{12}\right)$	$(-0.43, -2.0)$	11	$2.8\,\sigma$
$\left(\left[C_{Hq}^{(3)}\right]_{11}, [C_{Hud}]_{11}, [C_{Hud}]_{12}\right)$	$(0.27, -1.9, -2.4)$	16	$2.9\,\sigma$
$\left(\left[C_{Hq}^{(3)}\right]_{11}, \left[C_{Hq}^{(3)}\right]_{22}, [C_{Hud}]_{11}, [C_{Hud}]_{12}\right)$	$(0.59, 0.76, -2.6, -2.5)$	17	$2.9\,\sigma$
$\left(\left[C_{Hq}^{(3)}\right]_{11}, \left[C_{Hq}^{(1)}\right]_{11}, [C_{Hud}]_{11}, [C_{Hud}]_{12}\right)$	$(0.29, 0.11, -2.0, -2.4)$	13	$2.6\,\sigma$

Best pull

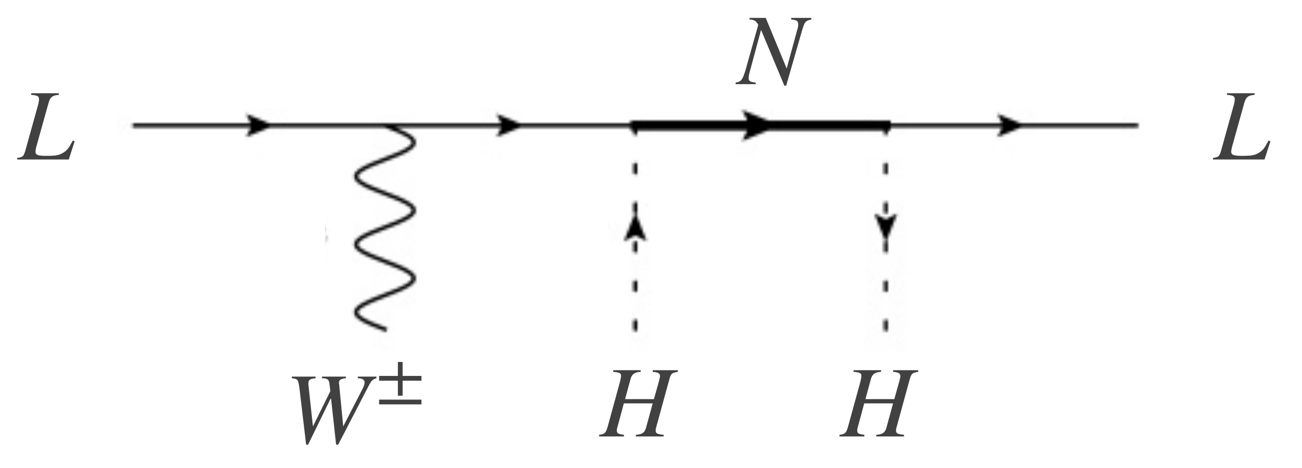
New physics interpretations of CAA

Heavy New Physics

- EFT fittings: $(H^\dagger i D^I_\mu H)(\bar{L}\gamma^\mu \tau^I L)$ fit [1912.08823]; **right-handed current fit** [1911.07821, 2112.02087];
  Best pull
  W-ℓ-ν fit [2002.07184]; G_F fit [2102.02825]
- Heavy SU(2)$_L$ vector boson (~10 TeV) [2005.13542]
- Leptoquark (~5TeV) [2104.05730]
- **Vector-like Quark (1-5 TeV)** [1906.02714, 2103.05549; 2001.02853; 2103.13409]
  Best pull
- Vector-like Lepton (1-2 TeV) [2005.03933; 2008.01113; 2008.03261]

Vector-like quark can explain CAA with EWPO, FCNC, collider bounds [Crivellin, Kirk, TK, Mescia, 2212.06862]

- Heavy right-handed neutrino (type I seesaw) **cannot explain** the tension
  [the unphysical region |mixing|^2 <0 is favored]



W-ℓ-ν coupling

Light NP

- MeV sterile neutrino [TK, Tobioka, 2308.13003]

Heavy NP: VL-quarks

- The most natural extension of the SM that leads to modified gauge couplings to quarks are the vector-like quarks (VLQs); theoretically well-motivated, e.g., by GUTs, composite and extra-dimensional models and little Higgs models
- Five kinds of VLQs that can provide the modified gauge coupling after integrated out

$$U : (\mathbf{3}, \mathbf{1}, {}^{2}\!/_{3}) \,, \quad D : (\mathbf{3}, \mathbf{1}, -{}^{1}\!/_{3}) \,, \quad Q : (\mathbf{3}, \mathbf{2}, {}^{1}\!/_{6}) \,, \quad T_1 : (\mathbf{3}, \mathbf{3}, -{}^{1}\!/_{3}) \,, \quad T_2 : (\mathbf{3}, \mathbf{3}, {}^{2}\!/_{3}) \,.$$

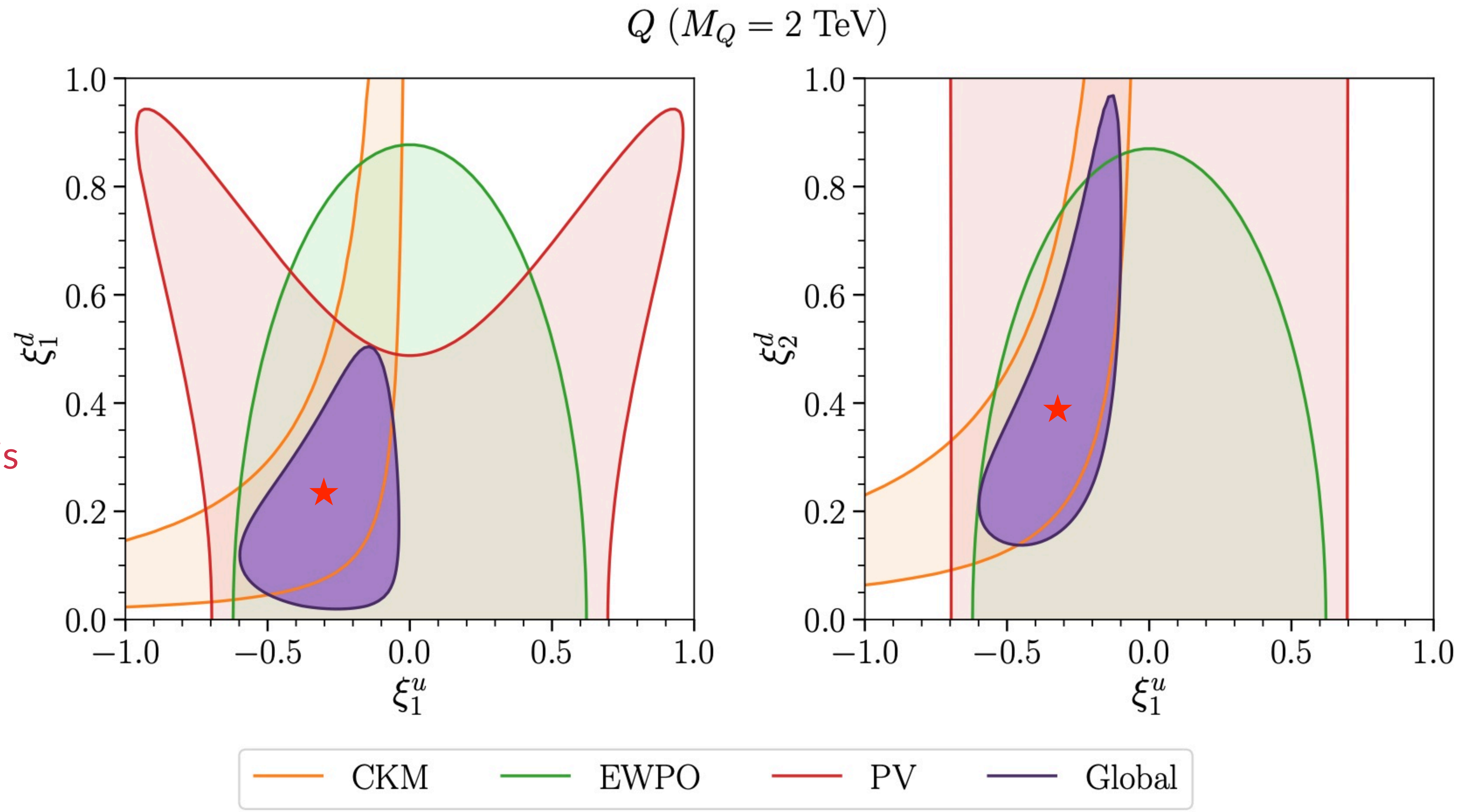
$C_{Hq}^{(3)}$

LH-W current

C_{Hud}

RH-W current (preferred)

Heavy NP: VL-quark Q



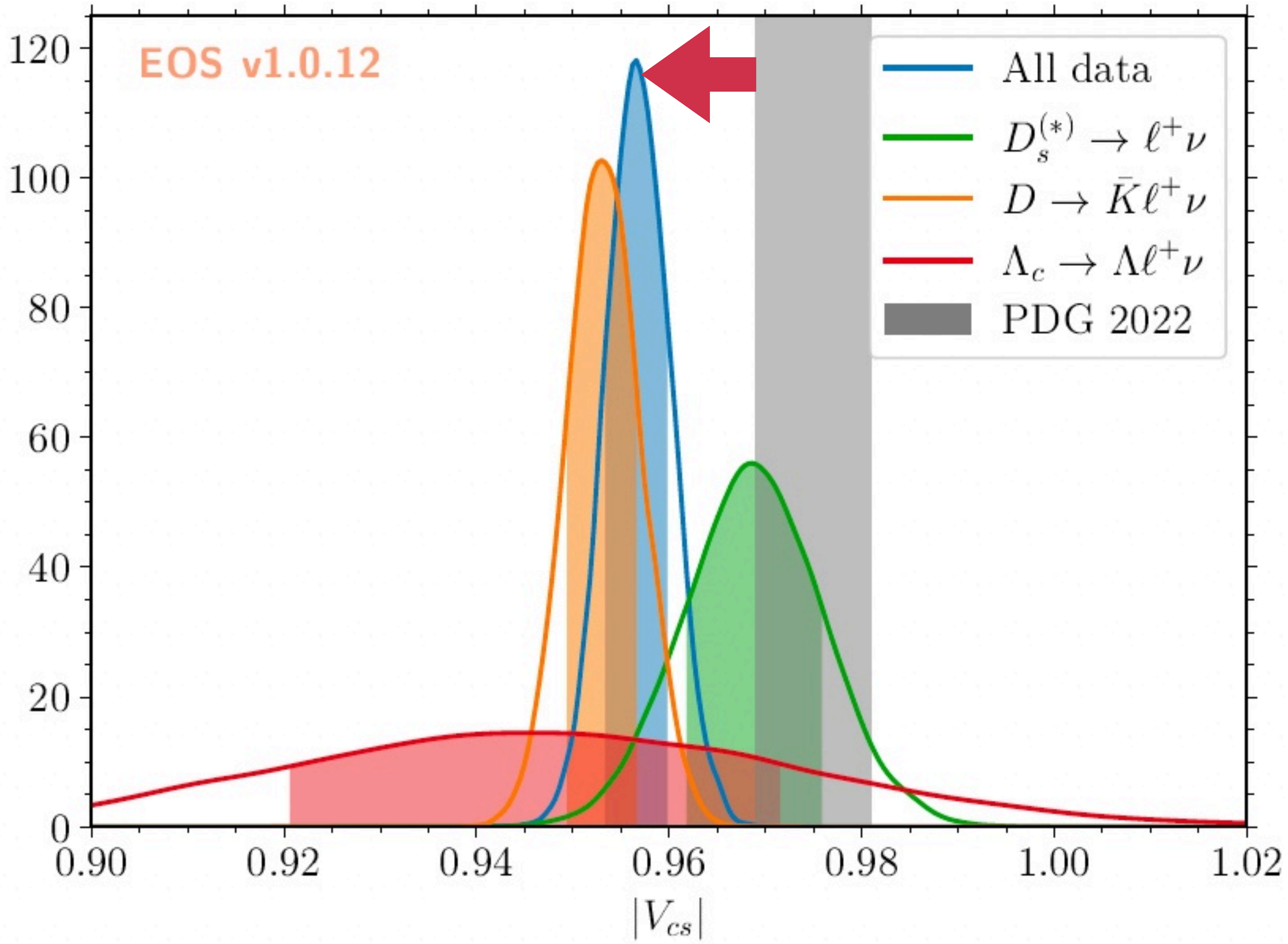
SM pull is 1.1σ

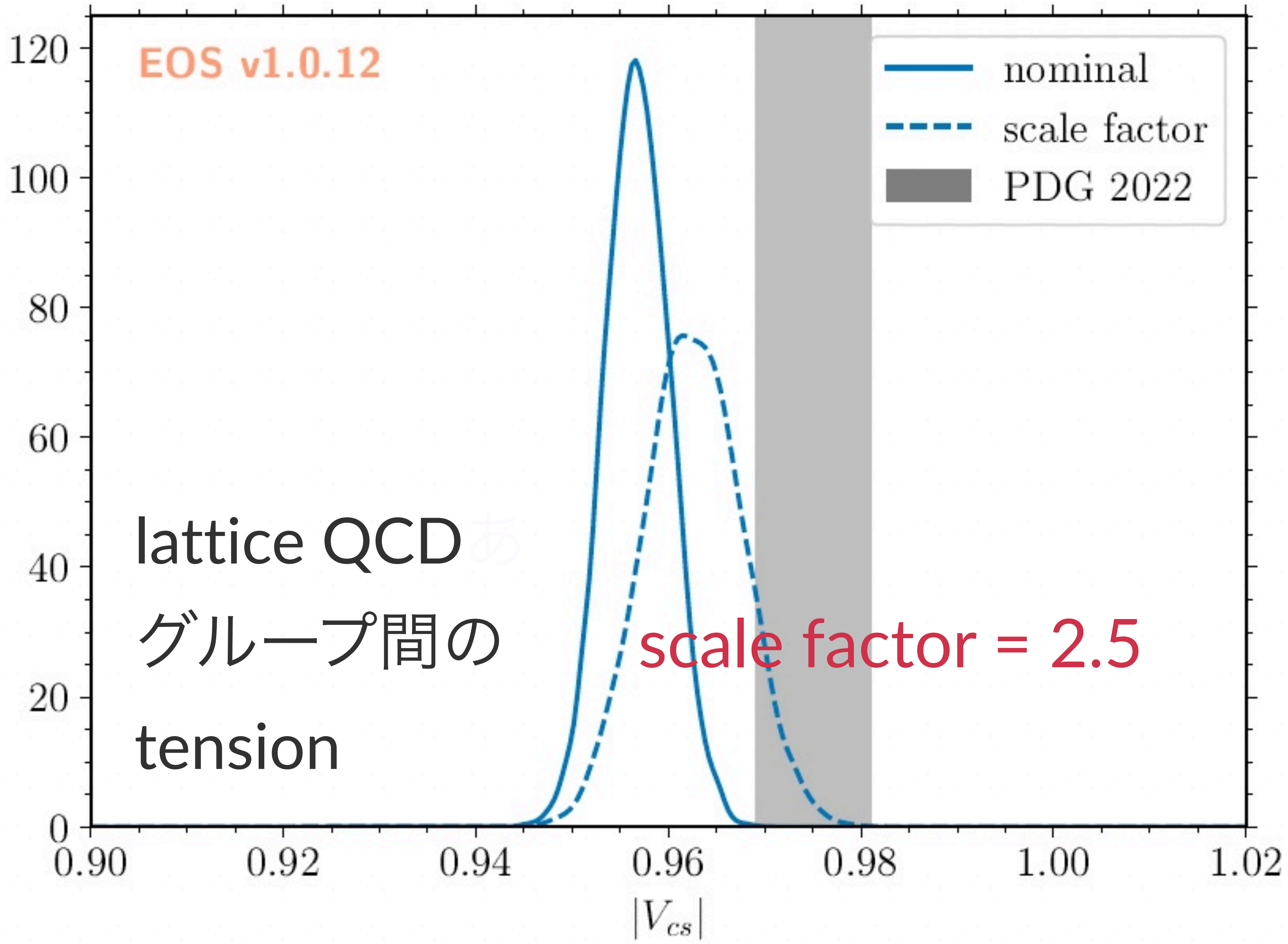
SM pull is 2.1σ

2nd-row unitarity

- Recently, 2nd-row unitarity violation was also reported [Bolognani, et al, 2407.06145]
- A global fit of $|V_{cs}|$ has been preformed

$D \to K$ FFs = FNAL/MILC, HPQCD (nominal)
= + ETM with scale factor





lattice QCD
グループ間の
tension

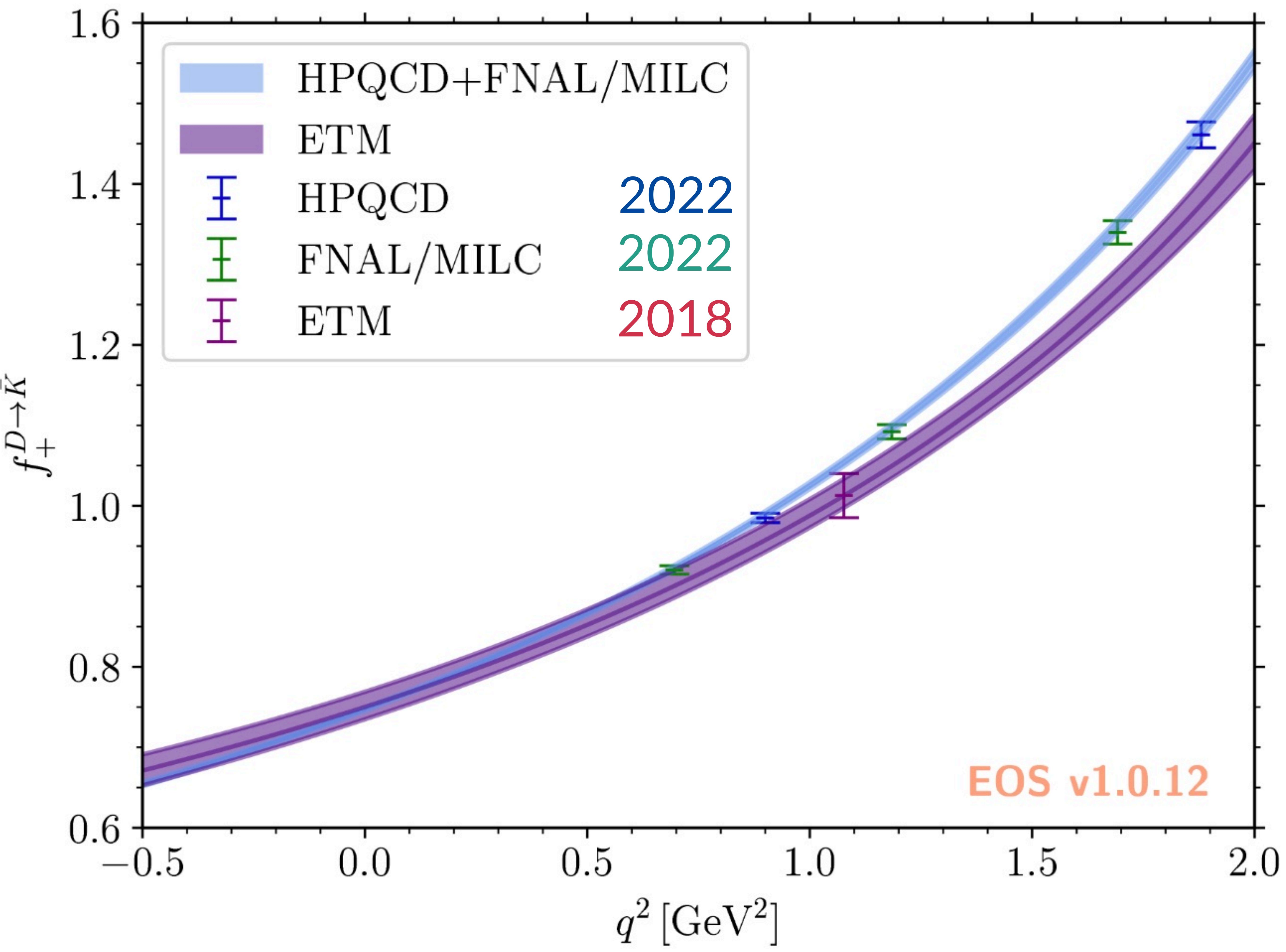
scale factor = 2.5

Different point from the PDG value

- Currently, |Vcs| is determined by $D^{0,+} \to K\ell^+\nu, D_s^+ \to \ell^+\nu$ decays; $|V_{cs}|^{\mathrm{PDG}} = 0.975(6)$
  - Including $D_s^{*+} \to e^+\nu, \Lambda_c \to \Lambda\ell^+\nu$ data
  - New BESIII data on $D_s^+ \to \mu^+\nu, D_s^+ \to \tau^+\nu$
  - New lattice results ($D \to K$)
  - Dispersive bounds are imposed in the form factors of $c \to s\ell^+\nu$ $(D \to K, \Lambda_c \to \Lambda)$
- New global fits: $|V_{cs}|^{\mathrm{nominal}} = 0.957(3)$ [-2.7σ from PDG], $|V_{cs}|^{\mathrm{with\,sf}} = 0.963(5)$ [-1.5σ]
- The difference mainly could be explained by (1) missing of the electroweak correction [photon/Z-W box] in PDG $D_s^+ \to \ell^+\nu$, (2) lattice FFs shift O(1)% in $D \to K$ [Bolognani, et al, 2407.06145]

D → K form factors f_+

$\langle \bar{K}(p') | \bar{s}\gamma_\mu c | D(p) \rangle$



O(1)% bad shift of |Vcs|

lattice QCDグループ間のtension

Other unitarity tests

Unitarity condition

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \boxed{V_{cd} \quad V_{cs} \quad V_{cb}} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \Rightarrow$$

2nd-row unitarity

nominal

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = \begin{cases} -0.034 \pm 0.008 \; [-4.3\sigma] \\ -0.022 \pm 0.012 \; [-1.9\sigma] \end{cases}$$

sf

[Bolognani, et al, 2407.06145]

1st-row unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2}^{\,10^{-5}} - 1 = -0.00151 \pm 0.00053 \; [-2.8\sigma]$$

[Crivellin, Kirk, TK, Mescia, 2212.06862]

Other unitarity tests

Unitarity condition

$$\begin{pmatrix} V_{ud} & \boxed{V_{us}} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

2nd-row unitarity

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = \begin{cases} -0.034 \pm 0.008 \; [-4.3\sigma] & \text{nominal} \\ -0.022 \pm 0.012 \; [-1.9\sigma] & \text{sf} \end{cases}$$

[Bolognani, et al, 2407.06145]

1st-row unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2}^{10^{-5}} - 1 = -0.00151 \pm 0.00053 \; [-2.8\sigma]$$

[Crivellin, Kirk, TK, Mescia, 2212.06862]

2nd-column unitarity

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 - 1 = \begin{cases} -0.032 \pm 0.006 \; [-5.2\sigma] & \text{nominal} \\ -0.021 \pm 0.010 \; [-2.0\sigma] & \text{sf} \end{cases}$$

Other unitarity tests

Unitarity condition

$$\begin{pmatrix} \boxed{V_{ud} \quad V_{us} \quad V_{ub}} \\ \boxed{V_{cd} \quad V_{cs} \quad V_{cb}} \\ V_{td} \quad V_{ts} \quad V_{tb} \end{pmatrix}$$

2nd-row unitarity

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = \begin{cases} -0.034 \pm 0.008 \; [-4.3\sigma] & \text{nominal} \\ -0.022 \pm 0.012 \; [-1.9\sigma] & \text{sf} \end{cases}$$

[Bolognani, et al, 2407.06145]

1st-row unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + \cancelto{10^{-5}}{|V_{ub}|^2} - 1 = -0.00151 \pm 0.00053 \; [-2.8\sigma]$$

[Crivellin, Kirk, TK, Mescia, 2212.06862]

2nd-column unitarity

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 - 1 = \begin{cases} -0.032 \pm 0.006 \; [-5.2\sigma] & \text{nominal} \\ -0.021 \pm 0.010 \; [-2.0\sigma] & \text{sf} \end{cases}$$

1st × 2nd-row unitarity

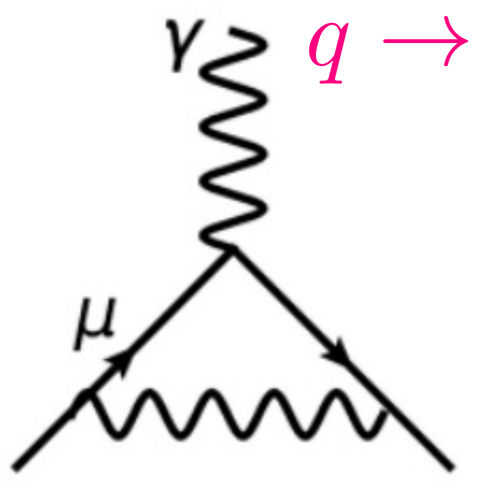
$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + \cancelto{10^{-4}}{V_{ub}V_{cb}^*} = 0$$

$$\approx -|V_{ud}||V_{cd}| + |V_{us}||V_{cs}| = \begin{cases} -(0.8 \pm 4.0) \times 10^{-3} & \text{nominal} \\ +(0.6 \pm 4.0) \times 10^{-3} & \text{sf} \end{cases}$$

ミューオンg-2 と 格子QCD

Muon magnetic dipole moment (*g*-2)

Definition of "spin *g*-factor" and g-2



$q \to 0$

$$\mathcal{L}_{\text{eff}} = -\frac{eQ_\ell}{4m_\ell}\bar{\ell}\sigma_{\mu\nu}\ell F^{\mu\nu} - \frac{eQ_\ell}{4m_\ell}a_\ell\bar{\ell}\sigma_{\mu\nu}\ell F^{\mu\nu} = -\frac{eQ_\ell}{8m_\ell}(2+2a_\ell)\bar{\ell}\sigma_{\mu\nu}\ell F^{\mu\nu}$$

EoM

$\mathcal{L} = \bar{\ell}(i\not{D} - m_\ell)\ell$ ["$F_1(0)$"]

UV-div. → renormalization

radiative corrections ["$F_2(0)$"]

UV-finite

spin *g*-factor $g_\ell = 2 + 2a_\ell$

g-2 : $a_\ell = \dfrac{g_\ell - 2}{2}$

$$\mathcal{H} = -\frac{eQ_\ell}{2m_\ell}g_\ell\vec{S}\cdot\vec{B} = -\vec{\mu}_\ell\cdot\vec{B}$$

rest frame

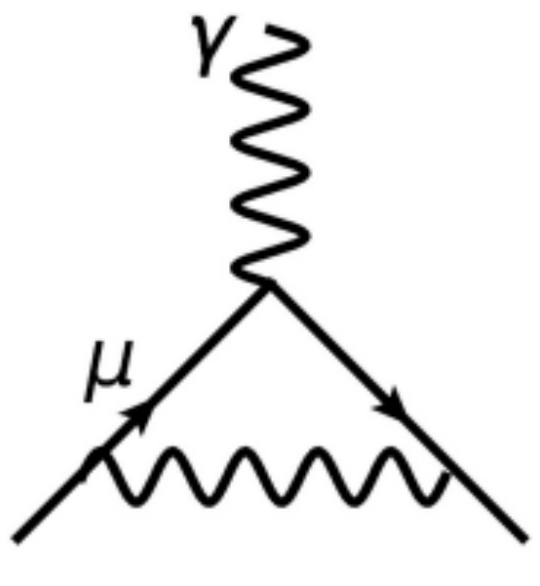
spin magnetic moment: $\vec{\mu}_\ell = g_\ell\dfrac{eQ_\ell}{2m_\ell}\vec{S}$

spin-magnetic field interaction

observable

Muon g-2

Theory (4 contributions)

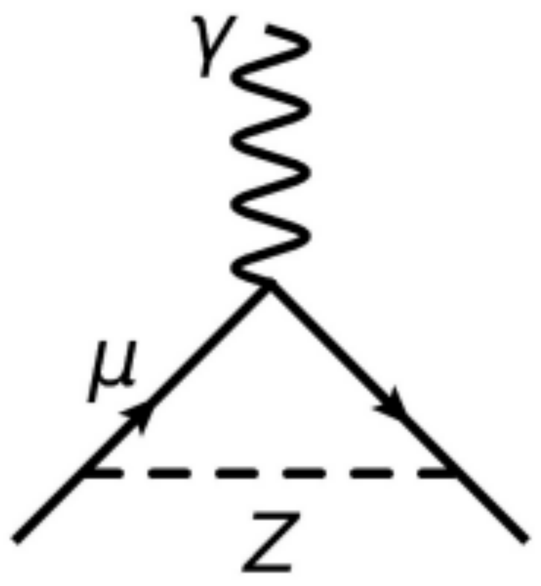


QED

4-loop analytic

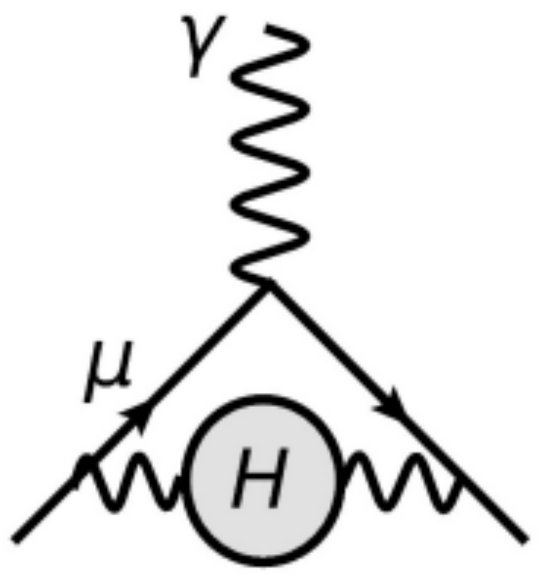
5-loop numeric

small disagreement here



EW

2-loop analytic

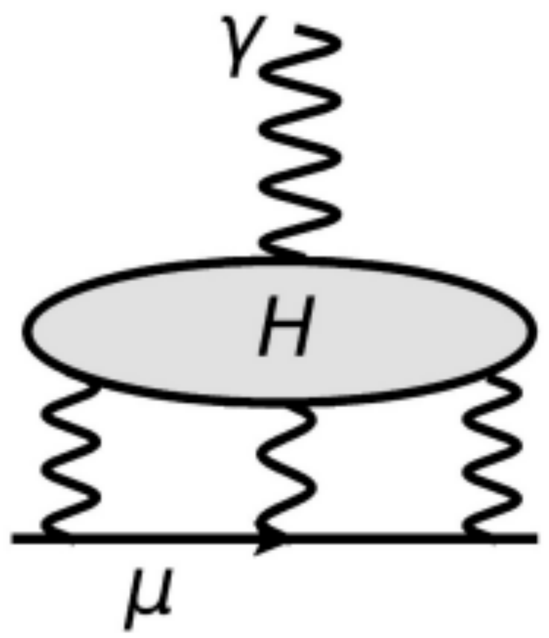


Hadronic vacuum polarization **(HVP)**

Phenomenological

Lattice

Discrepancy



Hadronic light-by-light **(HLbL)**

Pheno.

Lattice

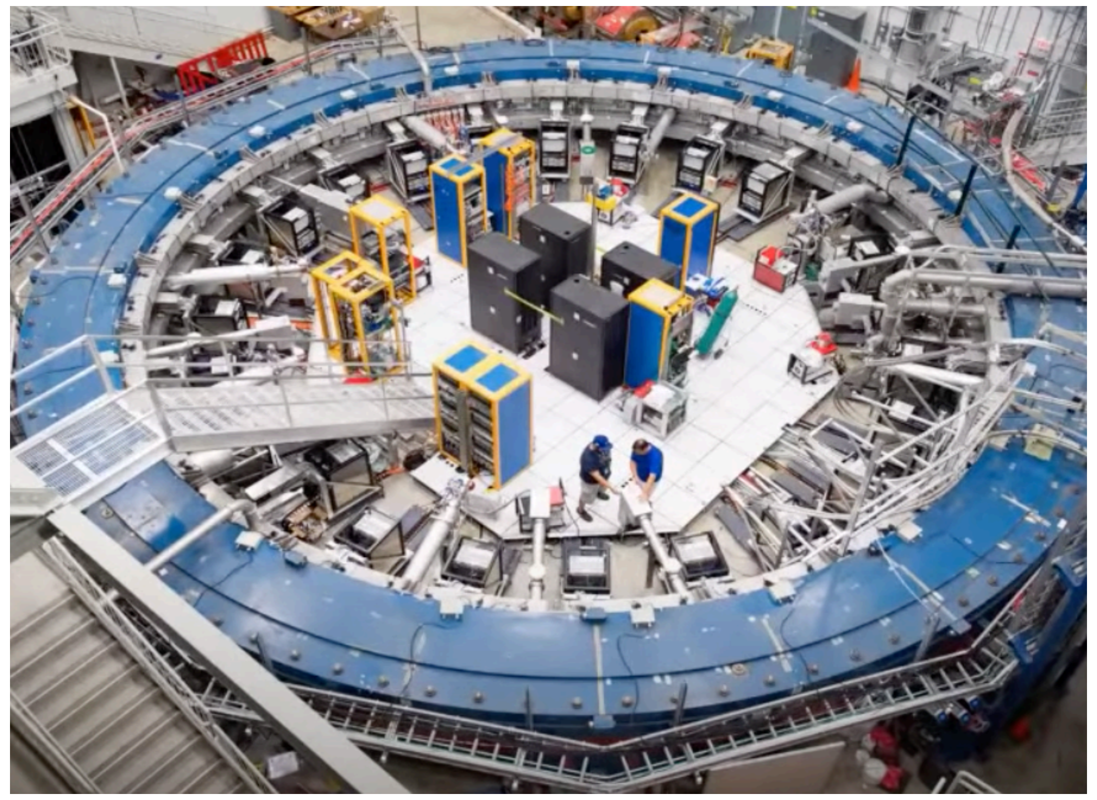
Consistent

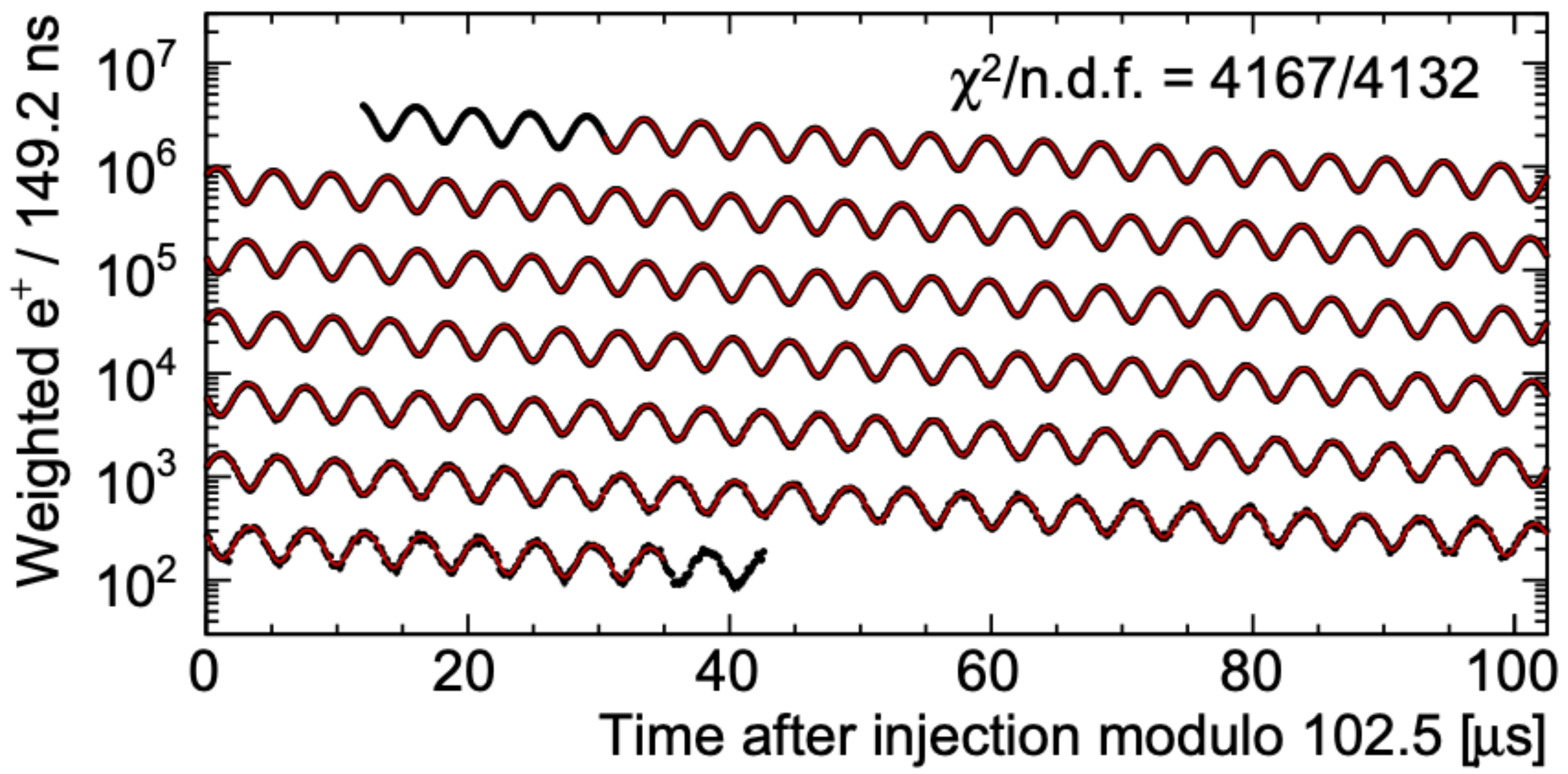
Experiment (2+1 exp.)

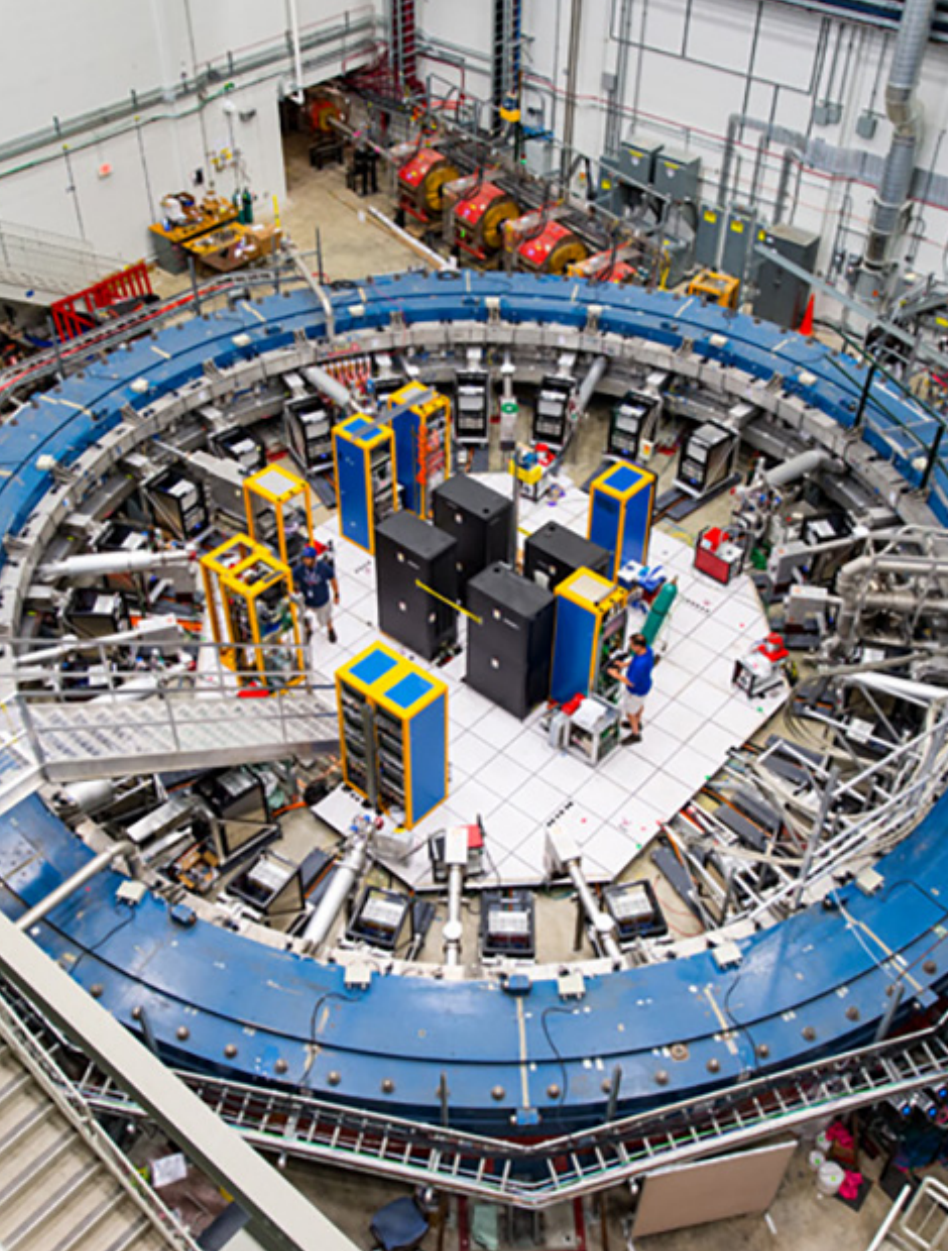
BNL ’97-‘01

FNAL **Run4** was done

J-PARC near future



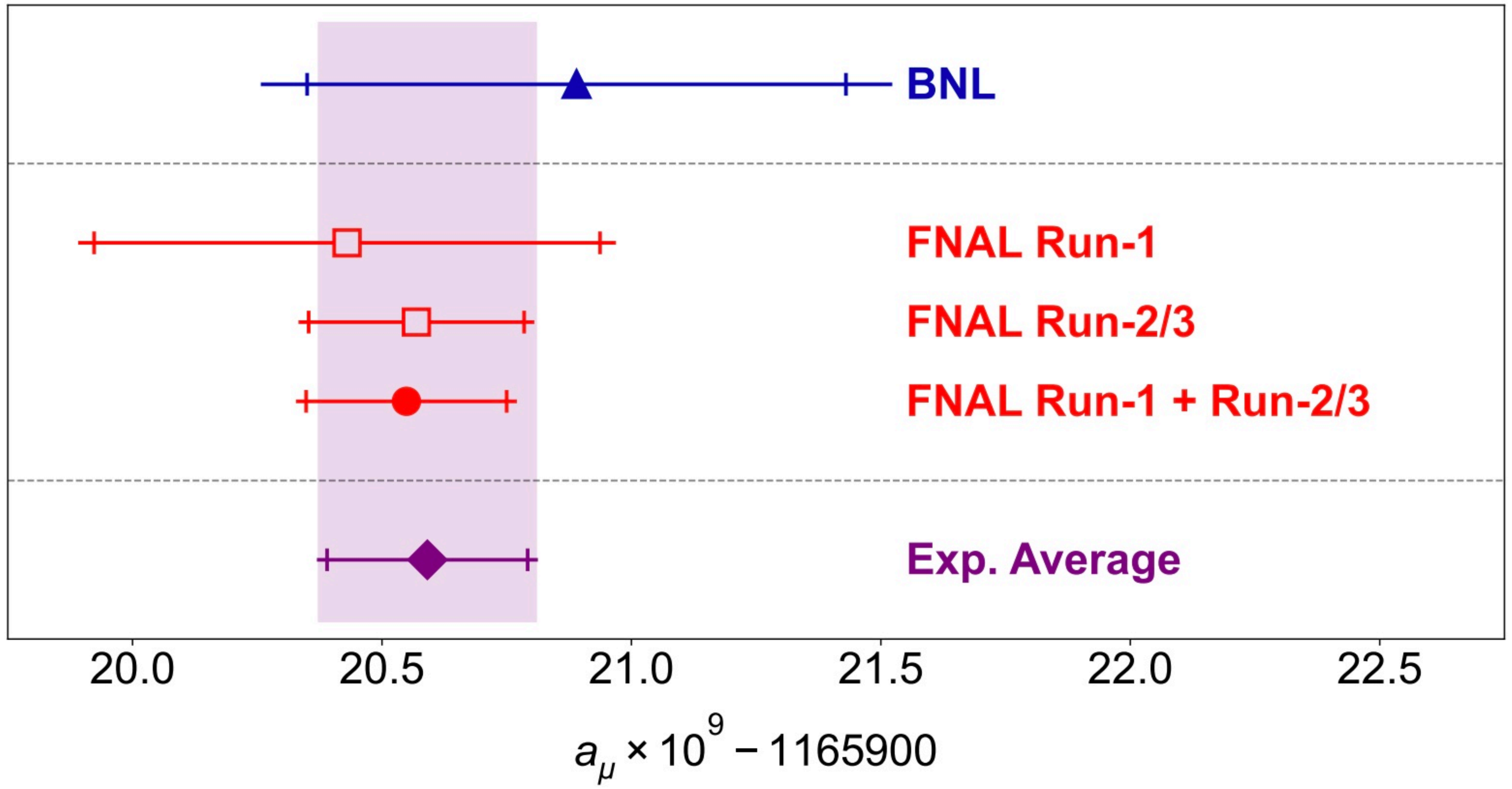




FNAL Run-2/3 data confirmed the previous results

Now, we know $a_\mu(\mathrm{Exp})$ very precisely

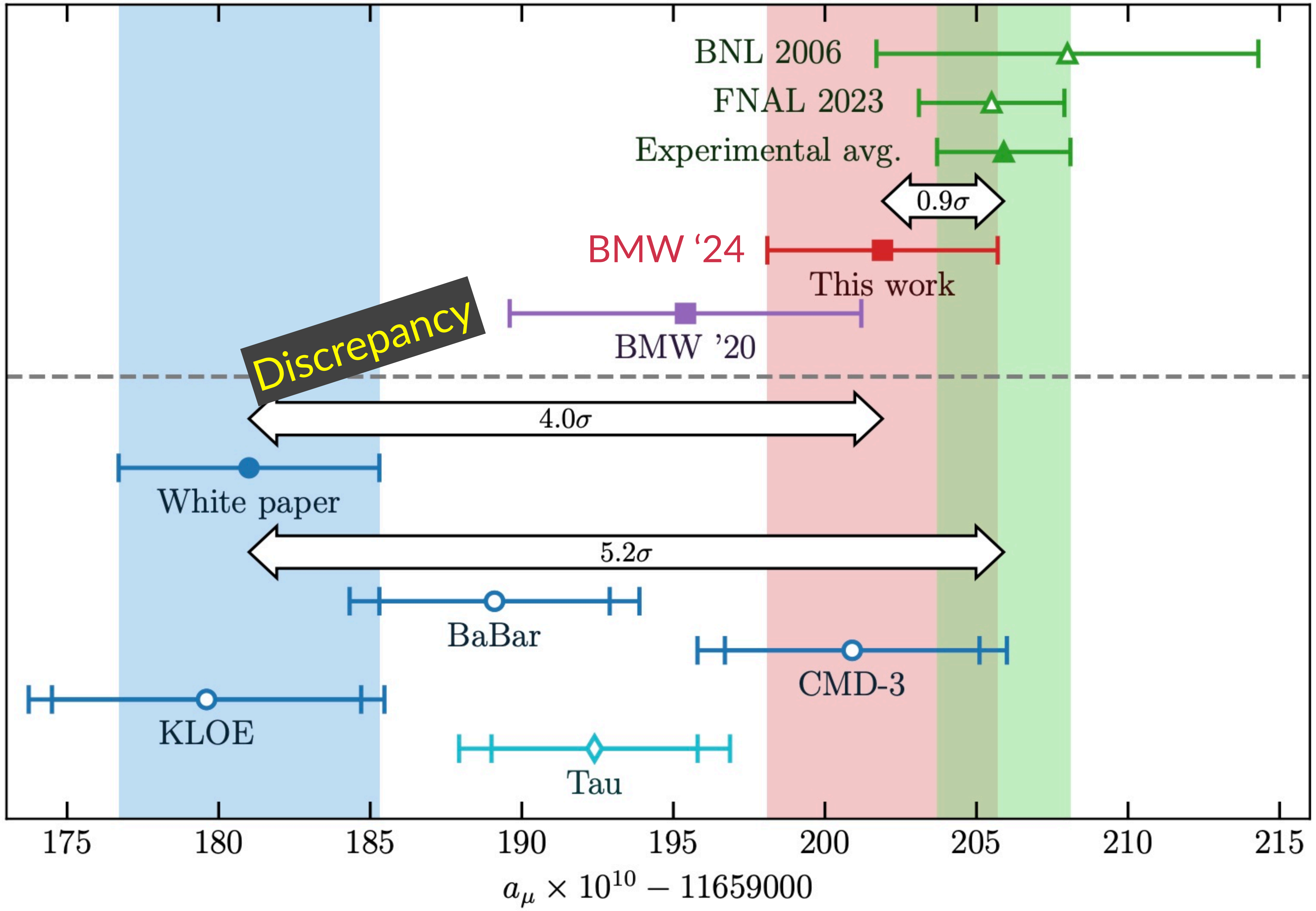
[Muon g-2 Collab. 2023, 2308.06230]

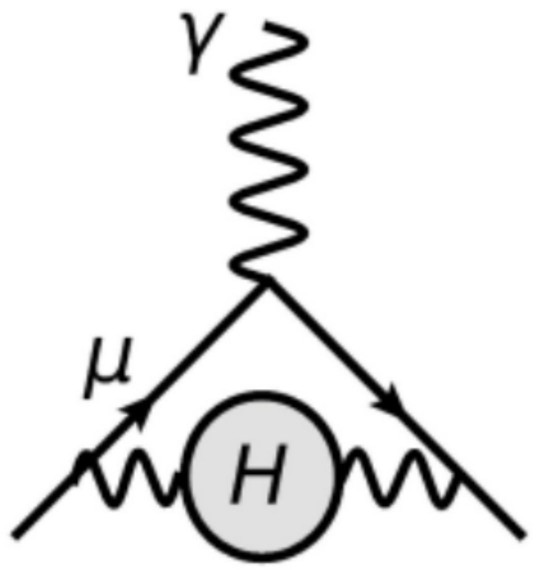


$$a_\mu(\mathrm{Exp}) = \frac{(g-2)_\mu}{2}(\mathrm{Exp}) = 116\,592\,059(22) \times 10^{-11}$$

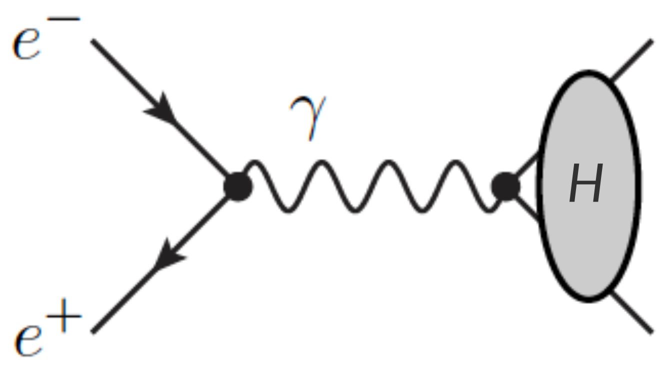
factor of two improved

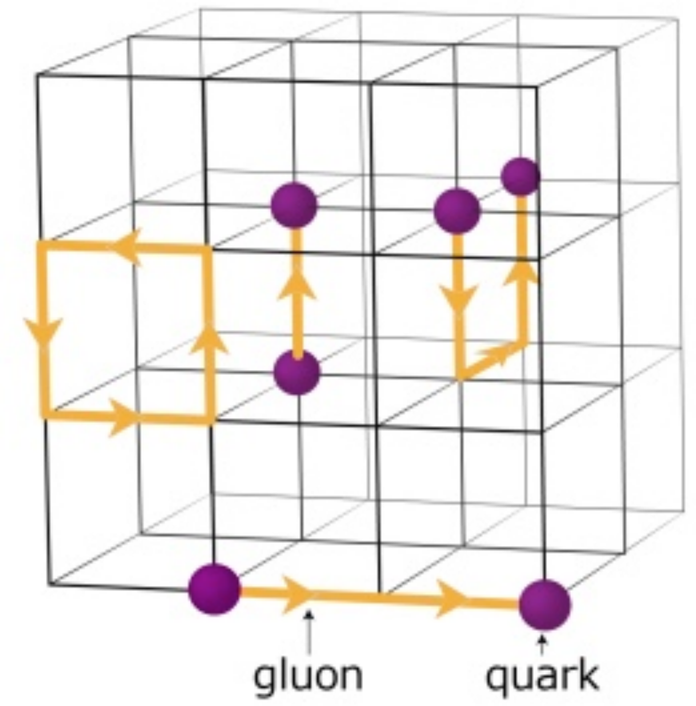
Current status of a_μ(Exp) vs a_μ(SM)





Hadronic-vacuum polarization (HVP)





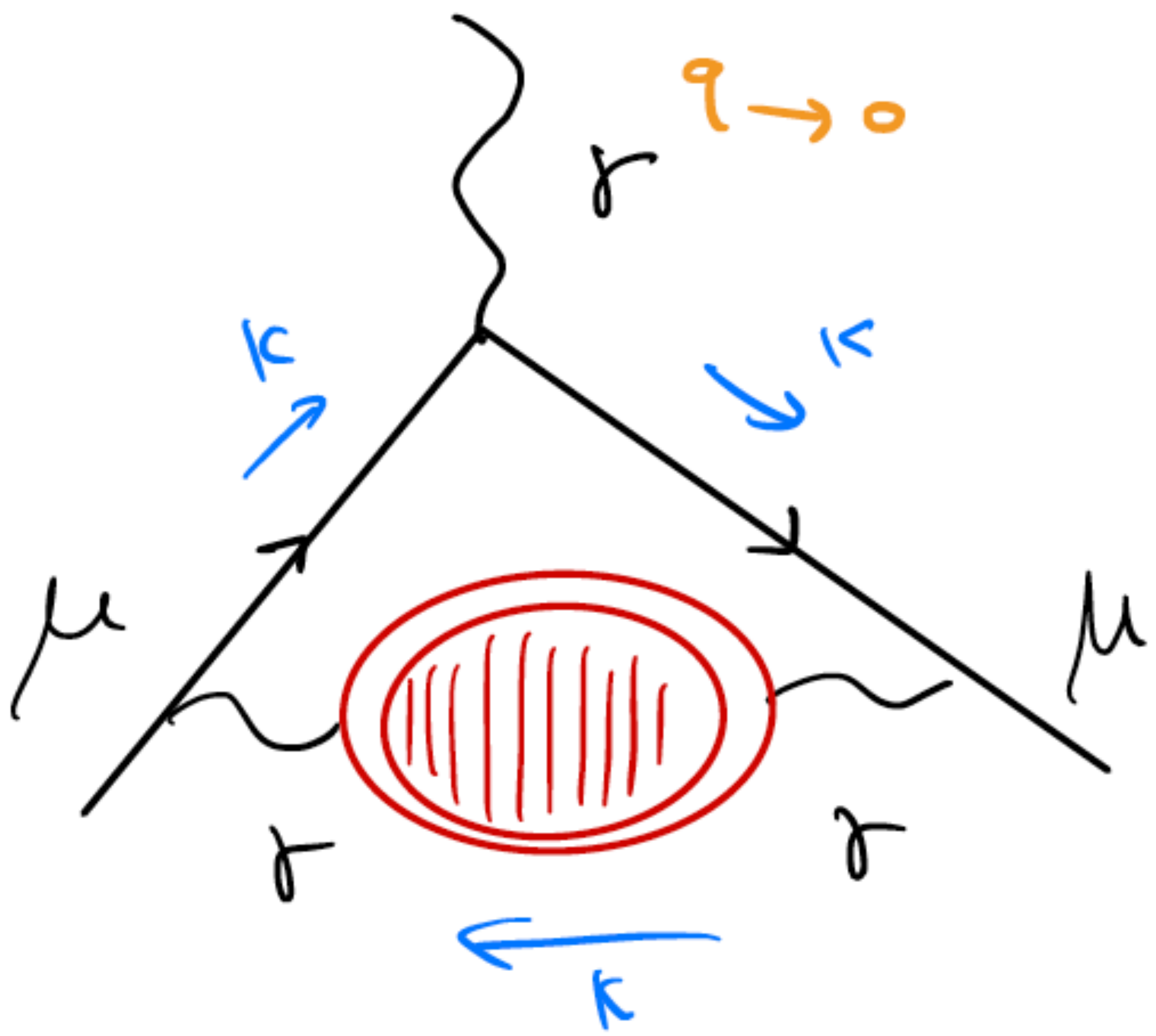
data-driven

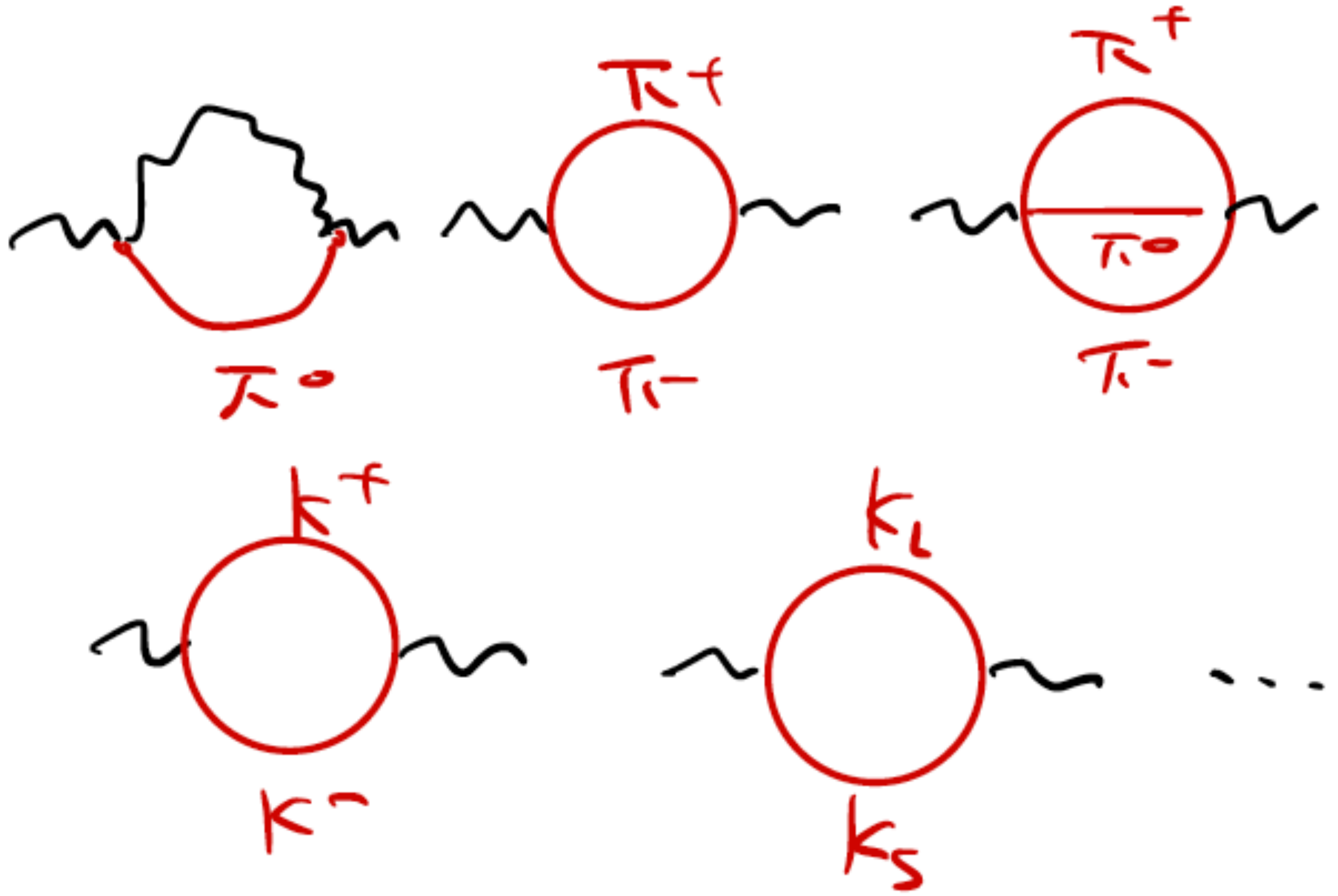
lattice from [Bi, et al]

[BMW, 2002.12347, 2407.10913]

leading-order HVP

- The virtual meson contributions to the vacuum polarization





$\pi^+\pi^-$	504.23(1.90)
$\pi^+\pi^-\pi^0$	46.63(94)
$\pi^+\pi^-\pi^+\pi^-$	13.99(19)
$\pi^+\pi^-\pi^0\pi^0$	18.15(74)
K^+K^-	23.00(22)
$K_S K_L$	13.04(19)
$\pi^0\gamma$	4.58(10)
Sum of the above	623.62(2.27)
$[1.8, 3.7]$ GeV (without $c\bar{c}$)	34.45(56)
J/ψ, $\psi(2S)$	7.84(19)
$[3.7, \infty)$ GeV	16.95(19)
Total $a_\mu^{\mathrm{HVP,\,LO}}$	692.8(2.4)

[White Paper, 2006.04822]

Data-driven approach (Dispersive approach) (1/2)

- General formula of the vacuum polarization (VP)

$$\vec{q} \quad = \quad \Pi^{\mu\nu}(q^2) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi(q^2)$$

- Dispersive relation of the HVP

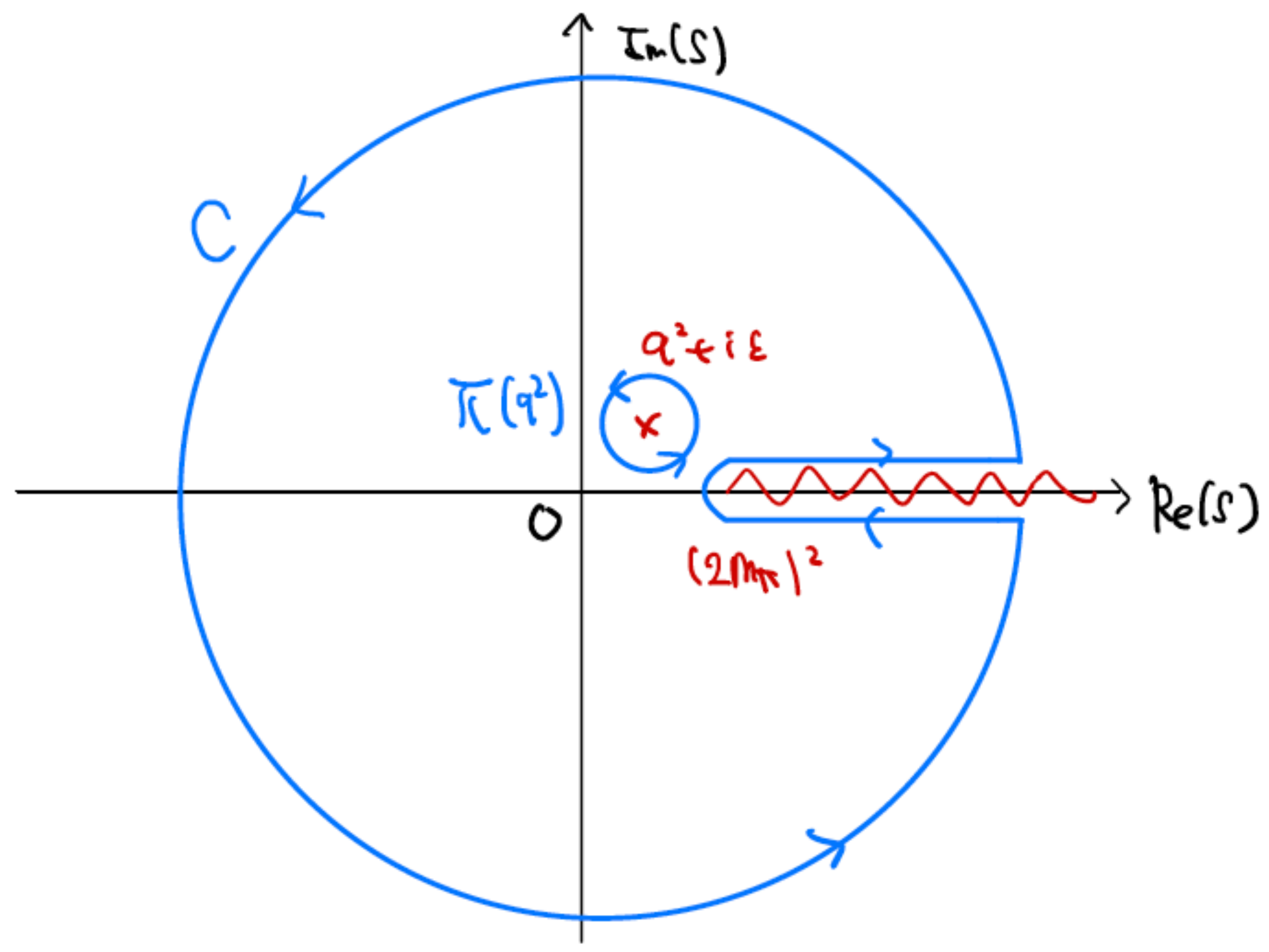
$$\Pi(q^2) = \int \frac{ds}{\pi(s-q^2)} \,\mathrm{Im}\, \underset{\text{had.}}{} (s)$$

- Optical theorem

$$2\,\mathrm{Im} \underset{\text{had.}}{} = \sum_{\text{had.}} \int d\Phi \left| \quad \right|^2$$

$$\sigma(e^+e^- \to \text{ hadron inclusive})(s)$$
$$= \sum_{\text{exc.}} \sigma(e^+e^- \to \text{ hadron exclusive})(s)$$

コーシーの積分公式



$$\sum_{\text{exc.}} \sigma(e^+e^- \to \text{ hadron exclusive})(s) \quad \vec{q} \quad =$$ LO-HVP

Data-driven approach (Dispersive approach) (2/2)

- Data-driven HVP contributions

$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha^2}{3\pi^2}\int ds \frac{R(s)K(s)}{s}$$

"R-ratio"

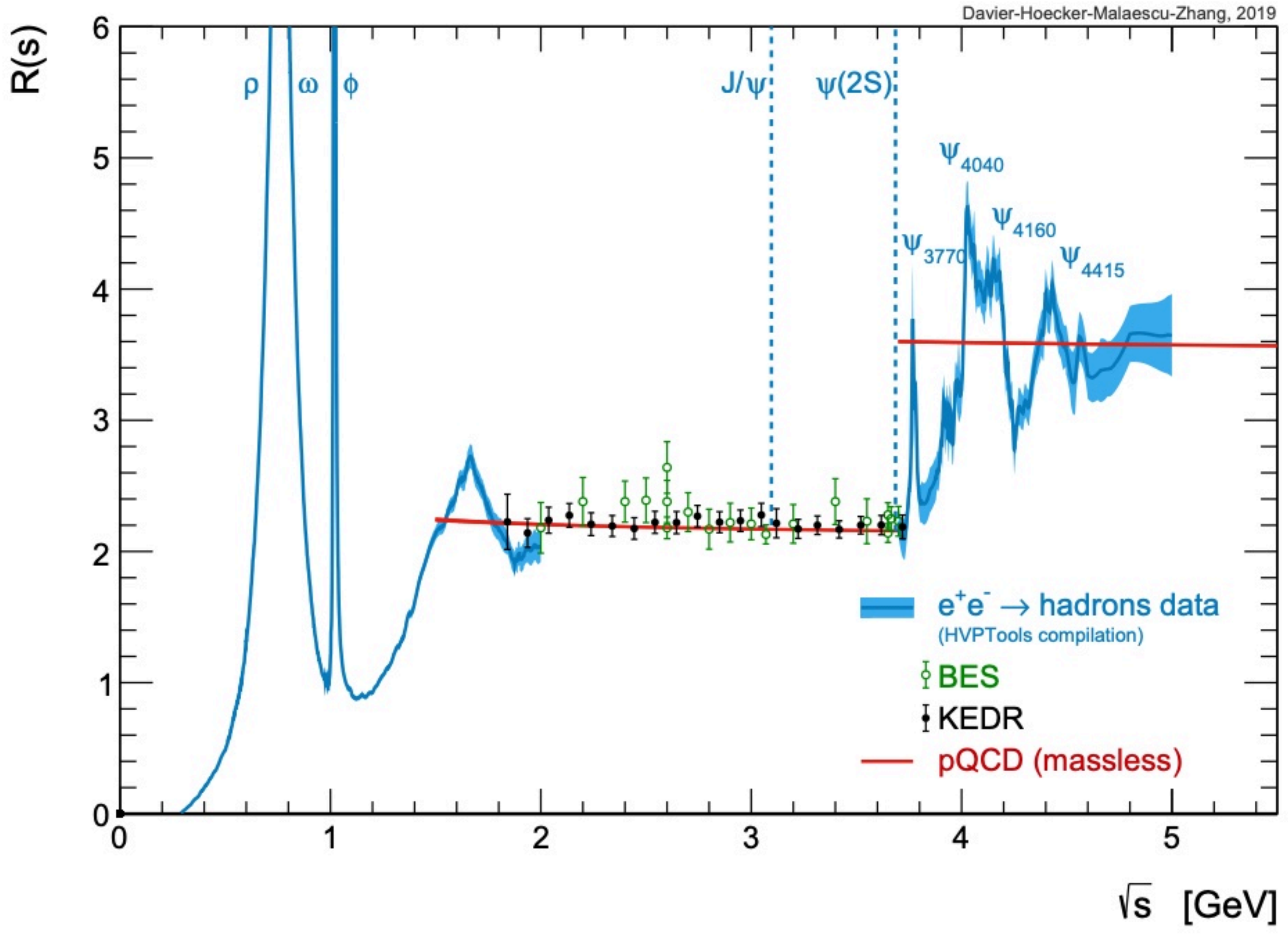
$$R(s) = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sigma(e^+e^- \to \mathrm{hadrons})\frac{3s}{4\pi\alpha^2}$$

"Kernel"

$$K(s, m_\mu^2) = \int_0^1 dx \frac{x(1-x)}{x^2 + (1-x)\frac{s}{m_\mu^2}}$$

$$\Rightarrow \quad a_\mu^{\mathrm{HVP,LO}} = 693.1(4.0) \times 10^{-10}$$

[White Paper, 2006.04822]



Lattice calculations (1/2)

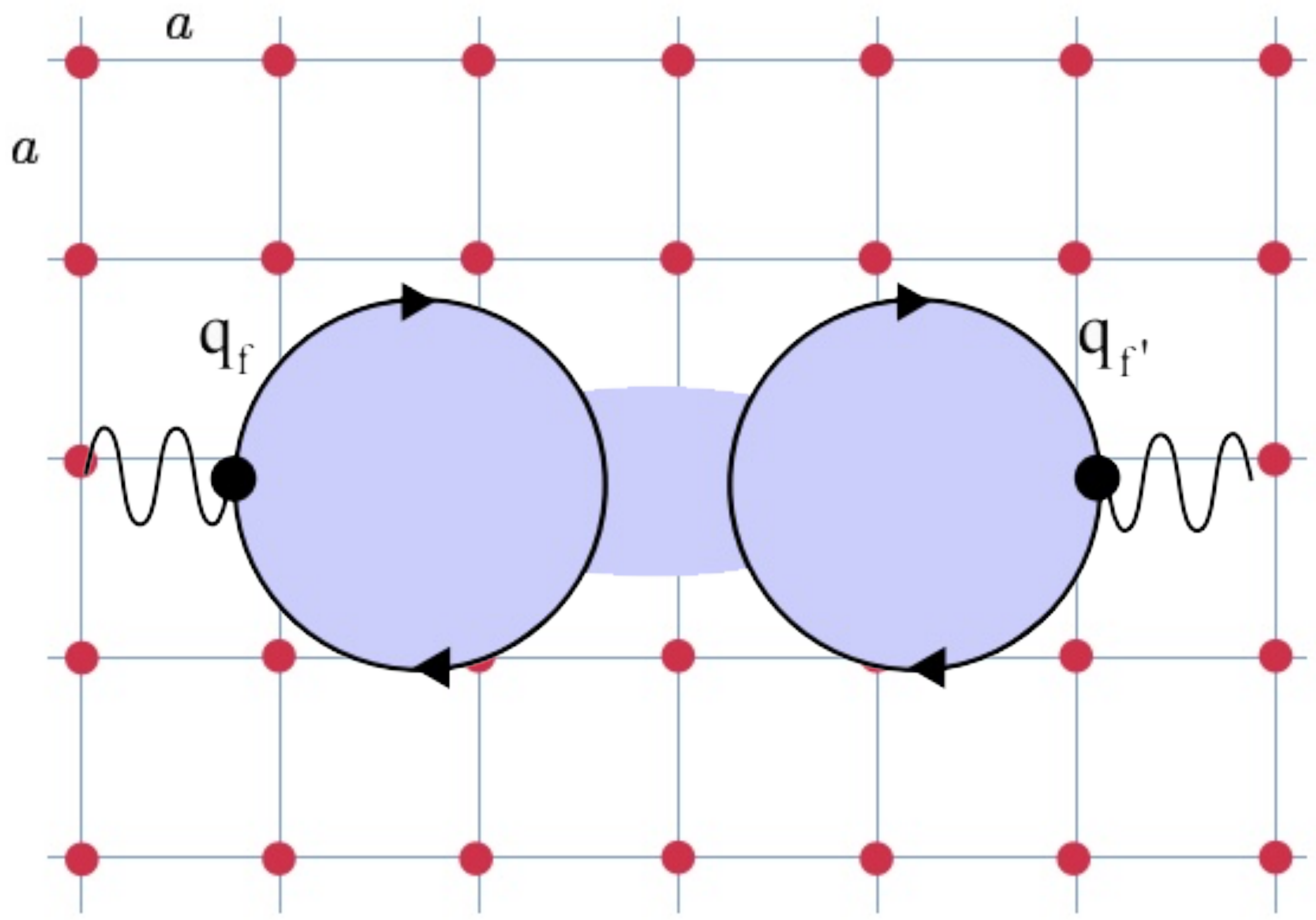
- On the other hand, the HVP on the lattice simulation can be obtained via the vector-current correlator (in Euclidian time *t*)

$$G(t) = \frac{1}{e^2} \sum_{\mu=1,2,3} \int d^3x \, \langle J_\mu(\vec{x}, t) J_\mu(0) \rangle$$

with $J_\mu = e\left(\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c\right)$

$$\Rightarrow \quad a_\mu^{\mathrm{HLO,LO}} = 4\alpha^2 m_\mu \int_0^\infty \mathrm{d}t G(t)\tilde{K}(t) t^3$$

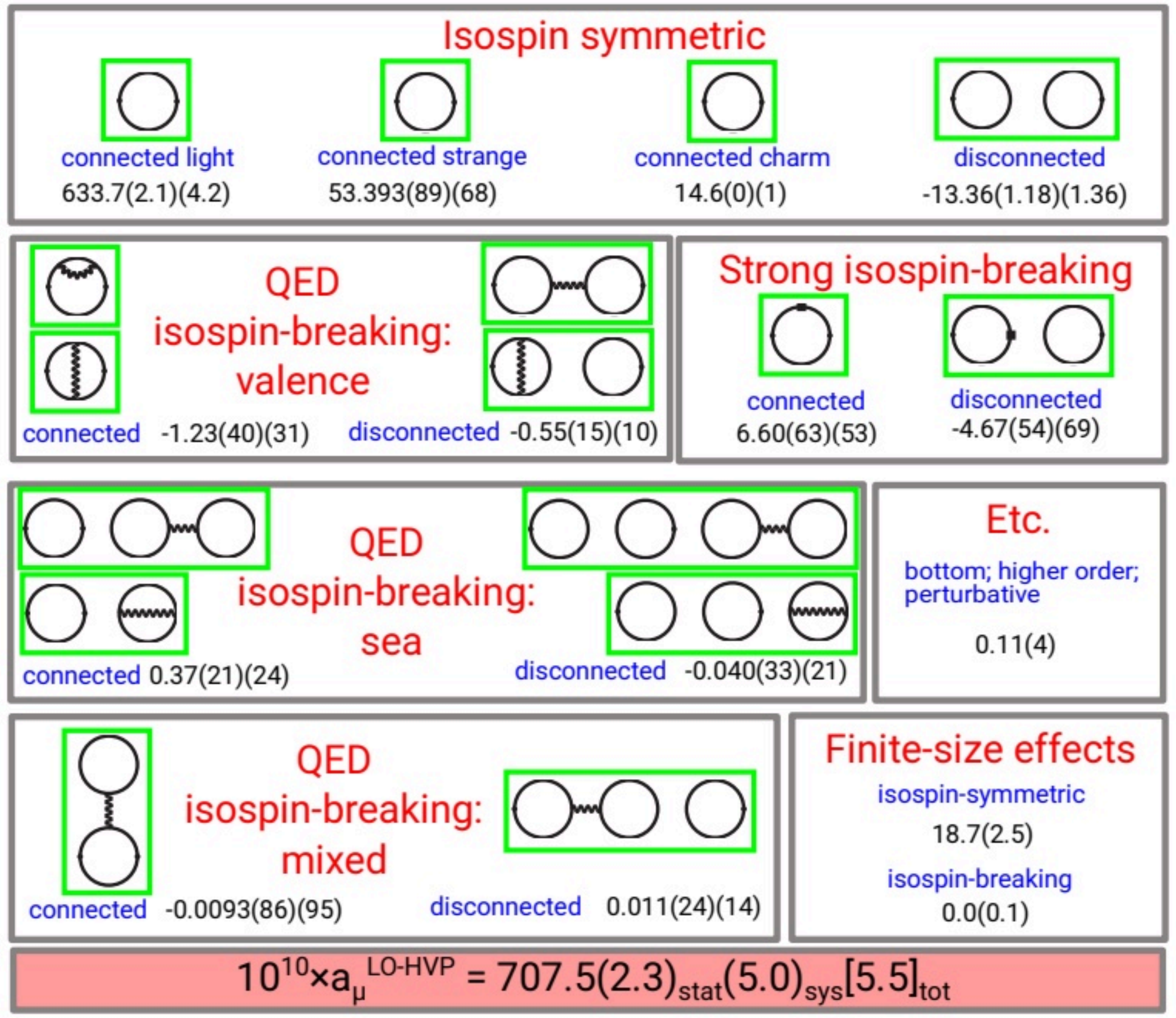
Kernel $\tilde{K}(t)$ is defined in, eg, 1107.4388



Lattice calculations (2/2)

- BMW group calculated all HVP by QCD and QED lattice simulation

  Nf=2+1+1, staggered quarks [BMW lattice, 2002.12347, Nature '21]



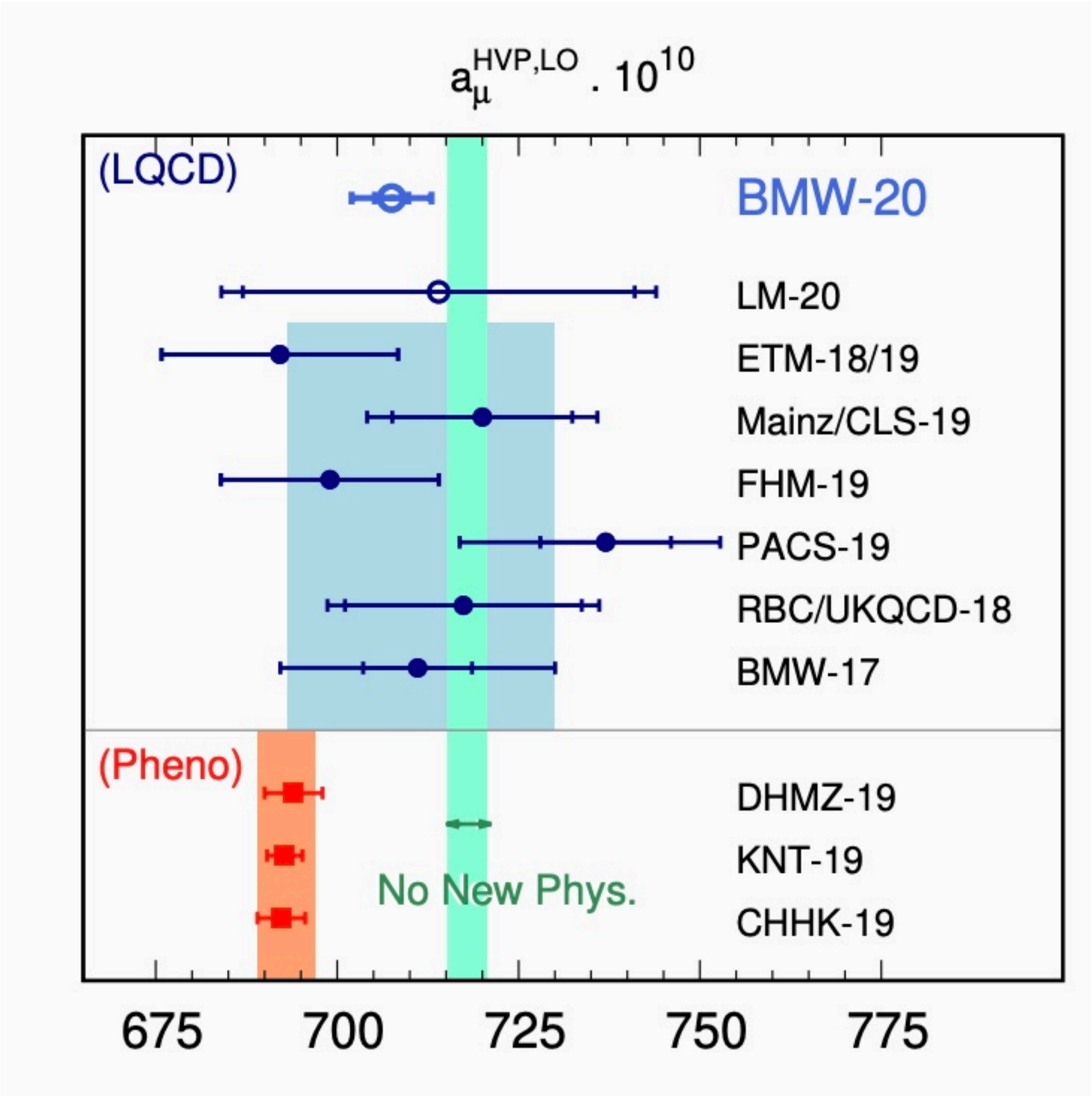


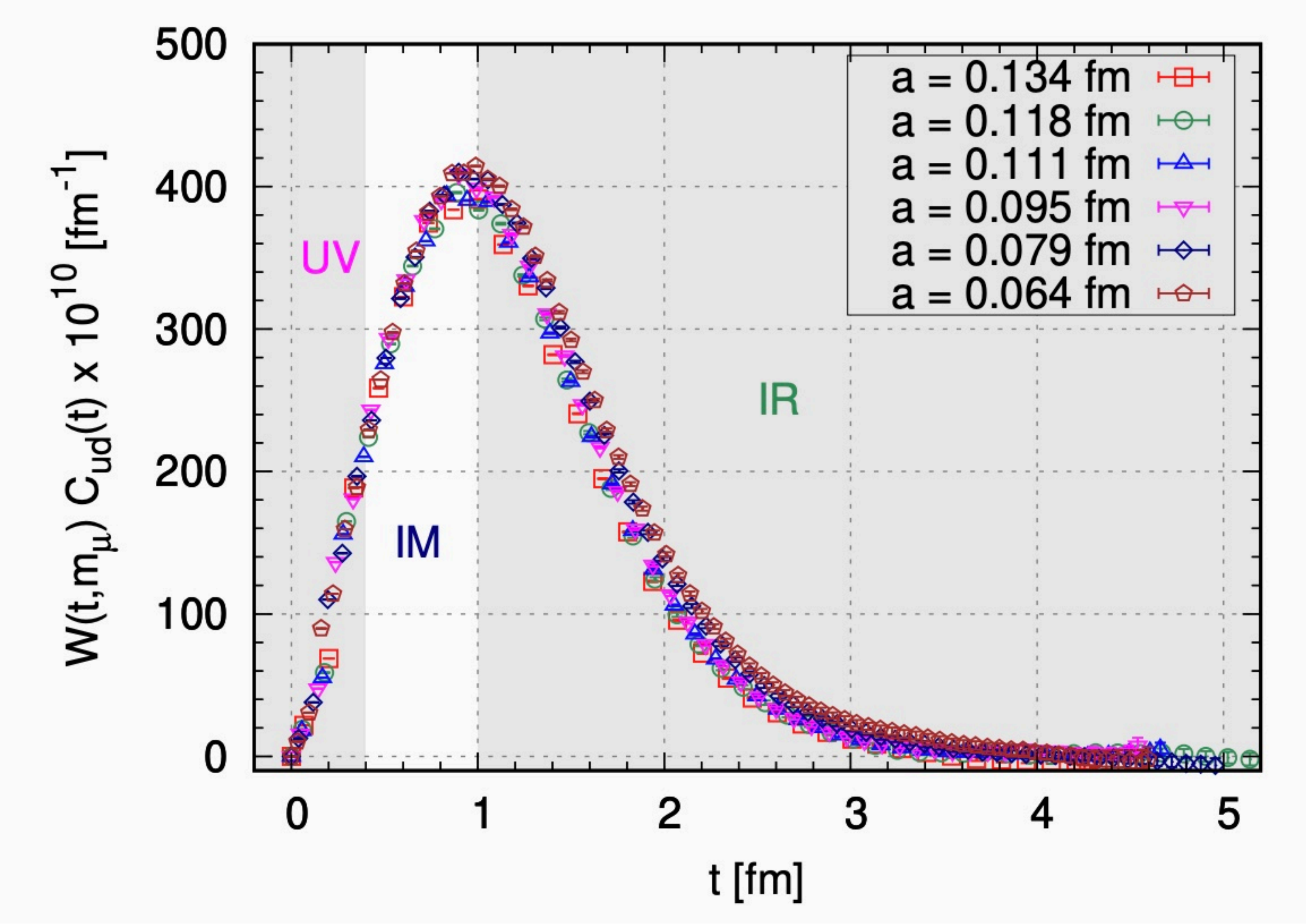
figure from Miura-san's slide

R-ratio value: $a_\mu^{\text{HVP,LO}} = 693.1(4.0)\times 10^{-10}$ 2.1σ and 4.0σ difference from BMW20 and BMW24

Windows method

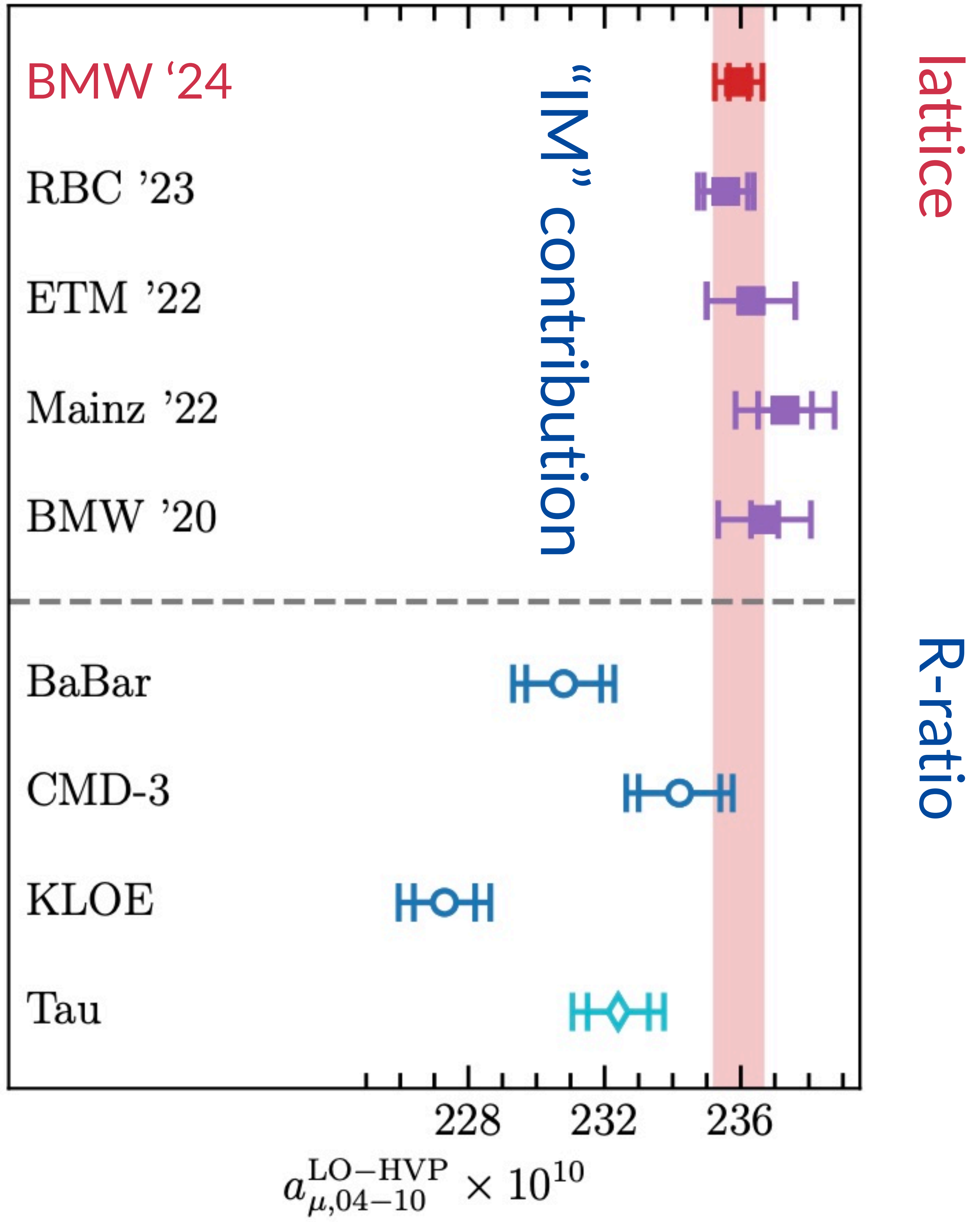
- Three other lattice groups confirmed the BMW result with "intermediate (IM) window methods" [RBC-UKQCD, 1801.07224]

$$a_\mu^{\mathrm{HVP}} = \int dt$$



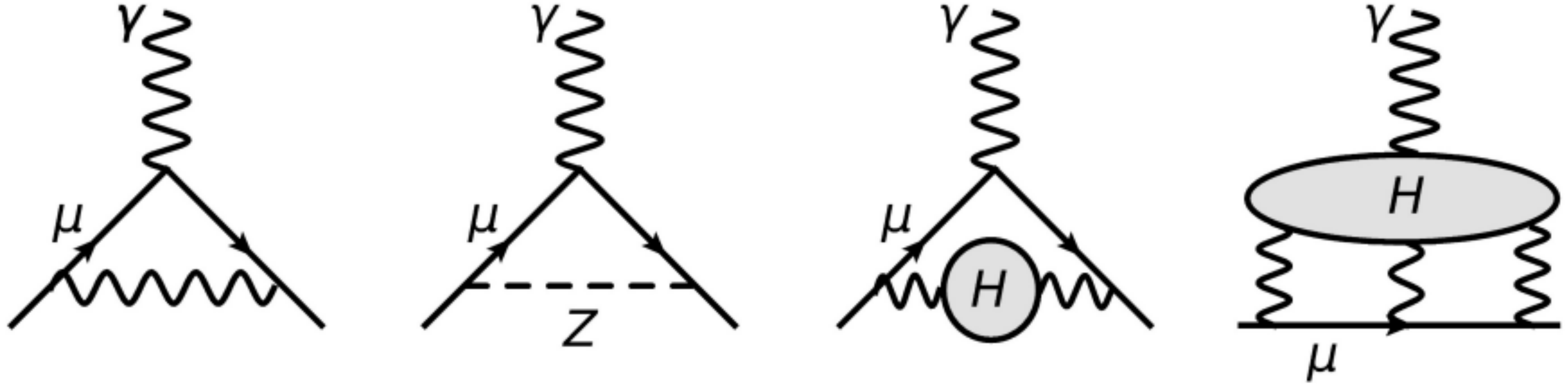
IMは全体の⅓

[BMW, 2407.10913]



New physics contributions

仮定: もし
格子QCD
に間違いが
あったら



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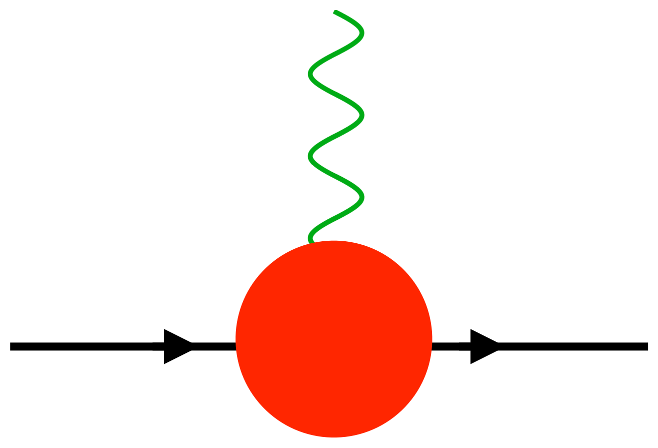


New physics (NP)

$$\Delta a_\mu \equiv a_\mu(\mathrm{Exp}) - a_\mu^{WP}(\mathrm{SM}) = (24.9 \pm 4.8) \times 10^{-10}\ (5.1\sigma)$$

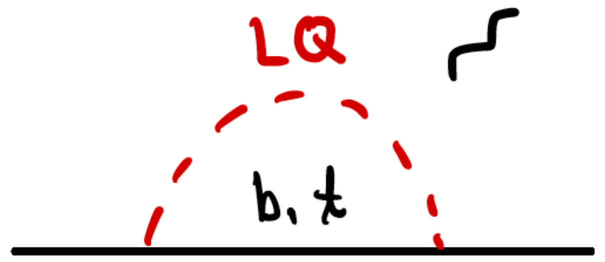
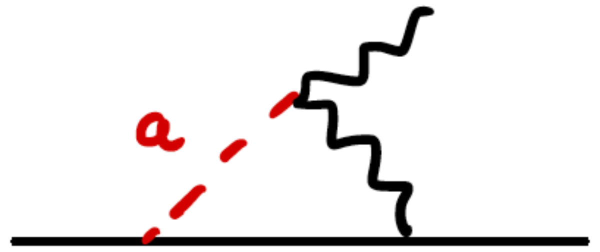
$$= \frac{m_\mu^2}{16\pi^2} \frac{g_{\mathrm{NP}}^2}{M_{\mathrm{NP}}^2}$$

$$M_{\mathrm{NP}} \approx g_{\mathrm{NP}} \times 150\,\mathrm{GeV}$$

- 5.1σ muon g-2 anomaly implies that NP scale is around the electroweak scale
- TeV scale NP (*e.g.*, **Supersymmetry**) with large g_{NP} (*e.g.*, $\tan\beta$, chiral enhancements)
- MeV scale NP with small g_{NP} (*e.g.*, $g_{\mathrm{NP}} \sim 10^{-3}$)

New physics interpretations

See [Endo, Iwamoto, TK, High Energy News, 2021] for details

NP type	diagrams	mass range	probe
Supersymmtery		200〜500 GeV	$\tilde{\chi}_2^0 \tilde{\chi}_1^{\pm} \to (h\tilde{\chi}_1^0)\,(W^{\pm}\tilde{\chi}_1^0)$ $pp \to \gamma\gamma \to \widetilde{\ell\ell^*}$
Leptoquark		1.5〜2.1 TeV	$pp \to \mathrm{LQ}\overline{\mathrm{LQ}}$ $Z \to \mu^+\mu^-$
Vector-like lepton		100 GeV〜1 TeV	$h \to \mu^+\mu^-$ $Z \to \mu^+\mu^-$
Scalar extensions		10〜100 GeV (A), 150〜300 GeV (H)	$Z \to \tau^+\tau^-$ $pp \to HA \to 4\tau$
Axion-like particle		40 MeV〜200 GeV	$e^+e^- \to \gamma a \to 3\gamma$
U(1) Lμ-Lτ		10〜200 MeV	$e^+e^- \to \mu^+\mu^- Z'$ $K \to \mu\nu Z'$, $\mu e \to \mu e Z'$

Heavy NP

Light NP

中性子EDM と 格子QCD

QCD θ項

- 対称性から許される Lagrangian

$$\mathcal{L}_{\rm SM} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\sum_i \bar{\psi}_i\gamma^\mu D_\mu\psi_i - \sum_{ij}\bar{\psi}_i y_{ij} H\psi_j + \text{h.c.}$$
$$+(D_\mu H)^\dagger(D^\mu H) - \mu^2|H|^2 - \lambda|H|^4 \quad +\bar{\theta}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad \text{with} \qquad \tilde{G}^{a\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}$$

- θ項はP (CP) 対称性を破る。サイズは特別な理由がなければO(1)のはず

- θ項は非摂動効果で中性子の電気双極子モーメント (EDM) $[d_n]$ を生む
  スピン-電場 相互作用
  - 格子QCDで計算可能

格子QCD上の中性子EDM

- 格子QCDではこの2点相関を計算

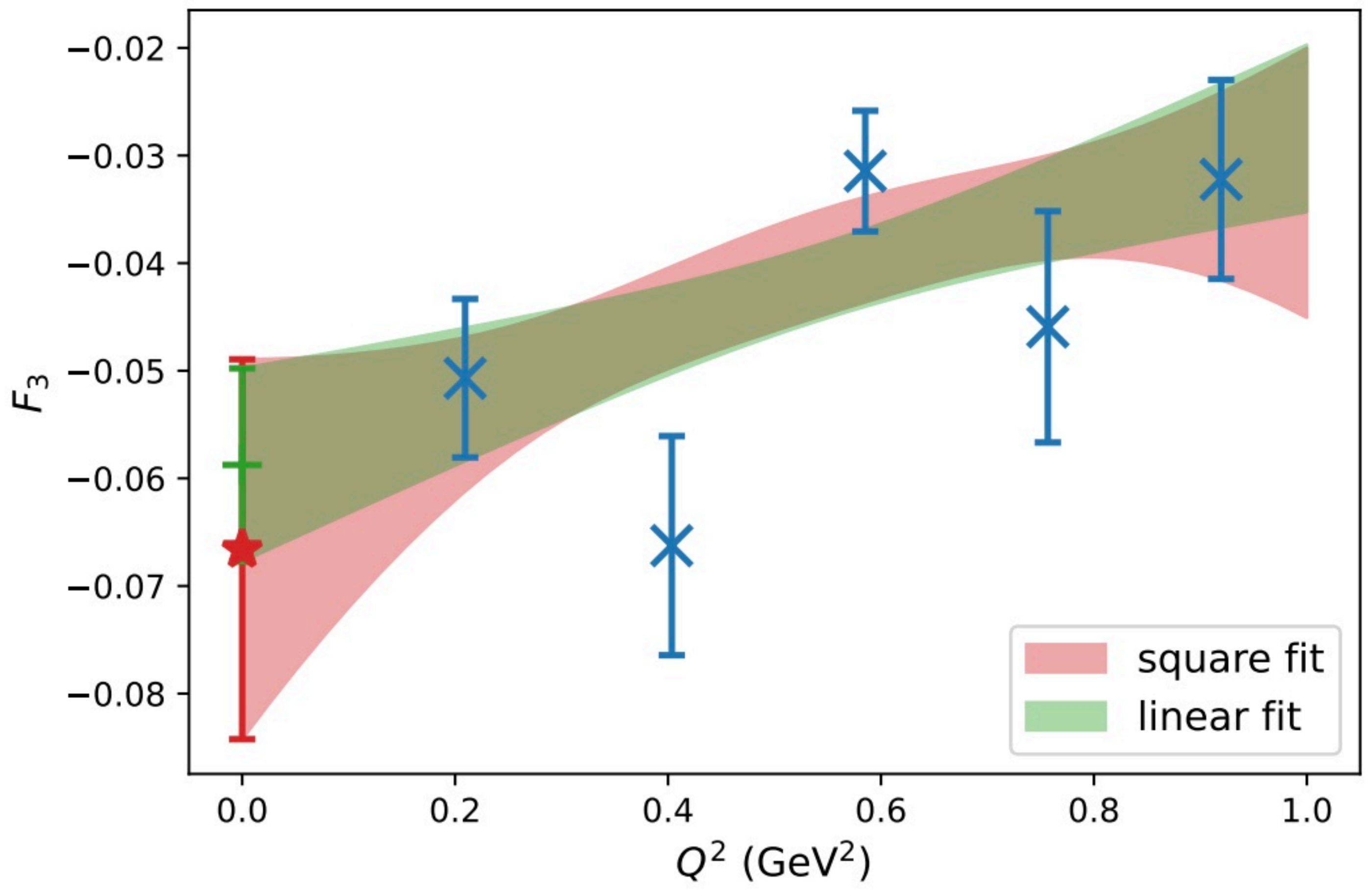
$$\langle N(p')\,|\bar{q}\gamma_\mu q|\,N(p)\rangle = \bar{u}^E(p')\left[\gamma_\mu^E F_1(q^2) - \sigma_{\mu\nu}^E q_\nu \frac{F_2(q^2)}{2m_N} - \sigma_{\mu\nu}^E q_\nu \gamma_5 \frac{F_3(q^2)}{2m_N}\right] u^E(p)$$

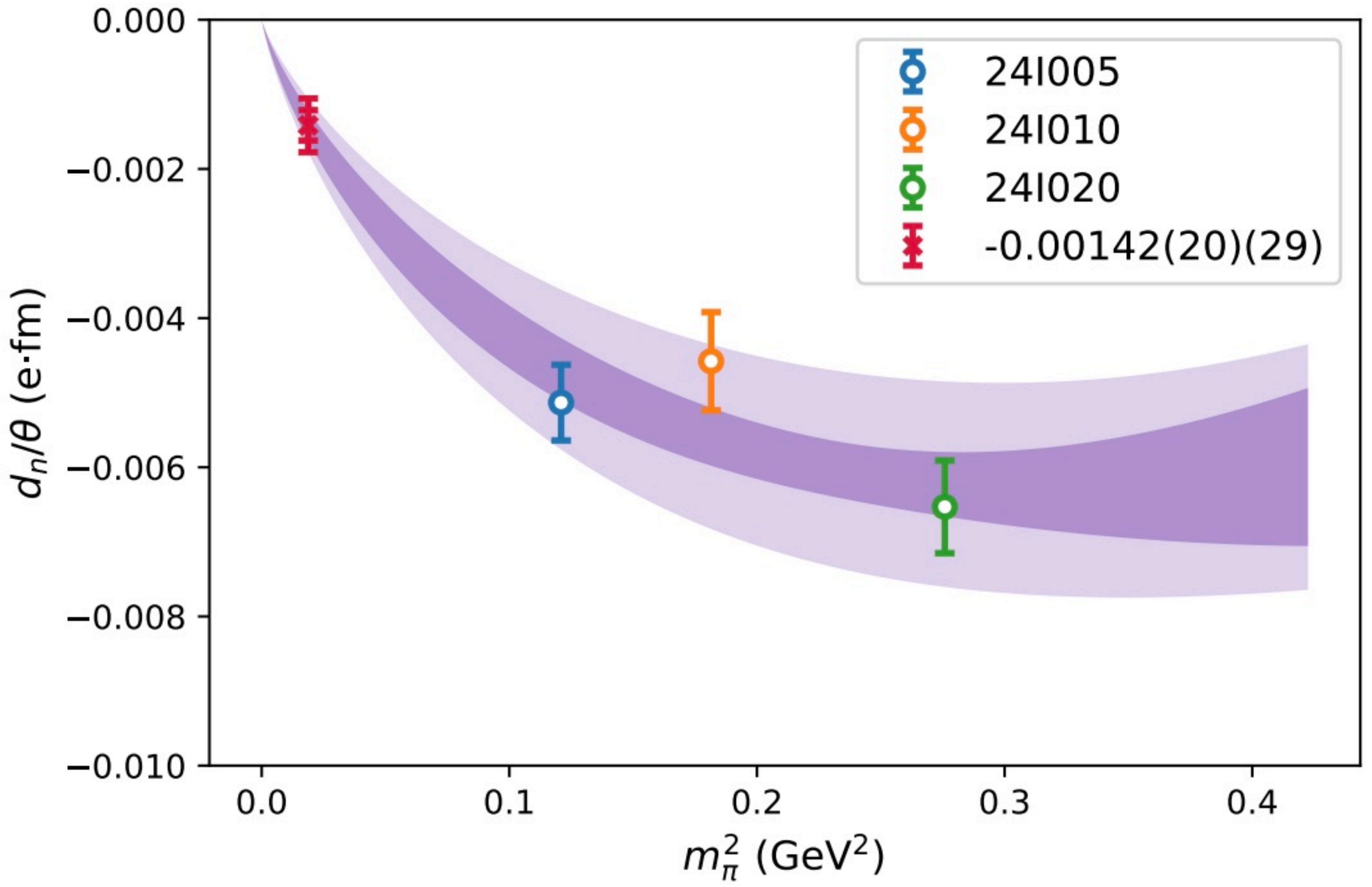
with $\exp(-S_\mathrm{E}) \to \exp\left(-S_\mathrm{E} - i\frac{\bar{\theta}}{(4\pi)^2}\int d^4x G_{\mu\nu}\tilde{G}^{\mu\nu}\right)$

[Liang, et al., 2301.04331]

➡ 中性子EDM

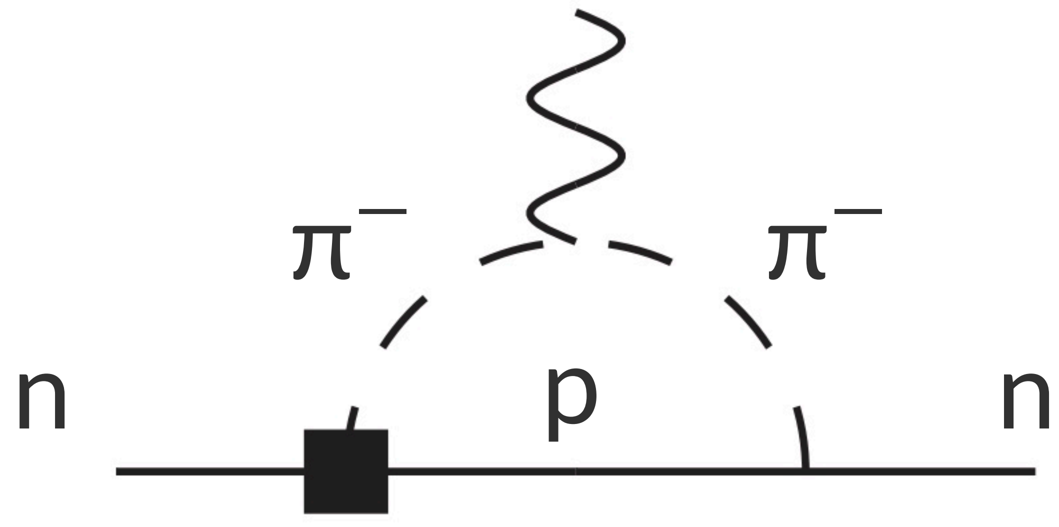
$$d_n = \frac{F_3(q^2\to 0)}{2m_N}\bar{\theta}$$





強いCP問題

LO ハドロニック ダイアグラム



- 格子QCDの最新の計算結果 [Liang, et al., 2301.04331]

$$d_n = -(1.48(34) \times 10^{-16})\,\bar{\theta}\; e\,\mathrm{cm}$$

- 一方で、中性子EDMの測定から、厳しい上限がついている [Abel et al., 2001.11966]

$$|d_n|_{\mathrm{exp}} < 1.8 \times 10^{-26}\; e\,\mathrm{cm}\;(90\,\%\,\mathrm{CL})$$

$$\Rightarrow \quad |\bar{\theta}| < 1.2 \times 10^{-10}\;(90\,\%\,\mathrm{CL})$$

- O(1)のはずの $\bar{\theta} < 10^{-10} \lll \delta_{\mathrm{CKM}} \simeq 1.2$ ⇒ 強いCP問題

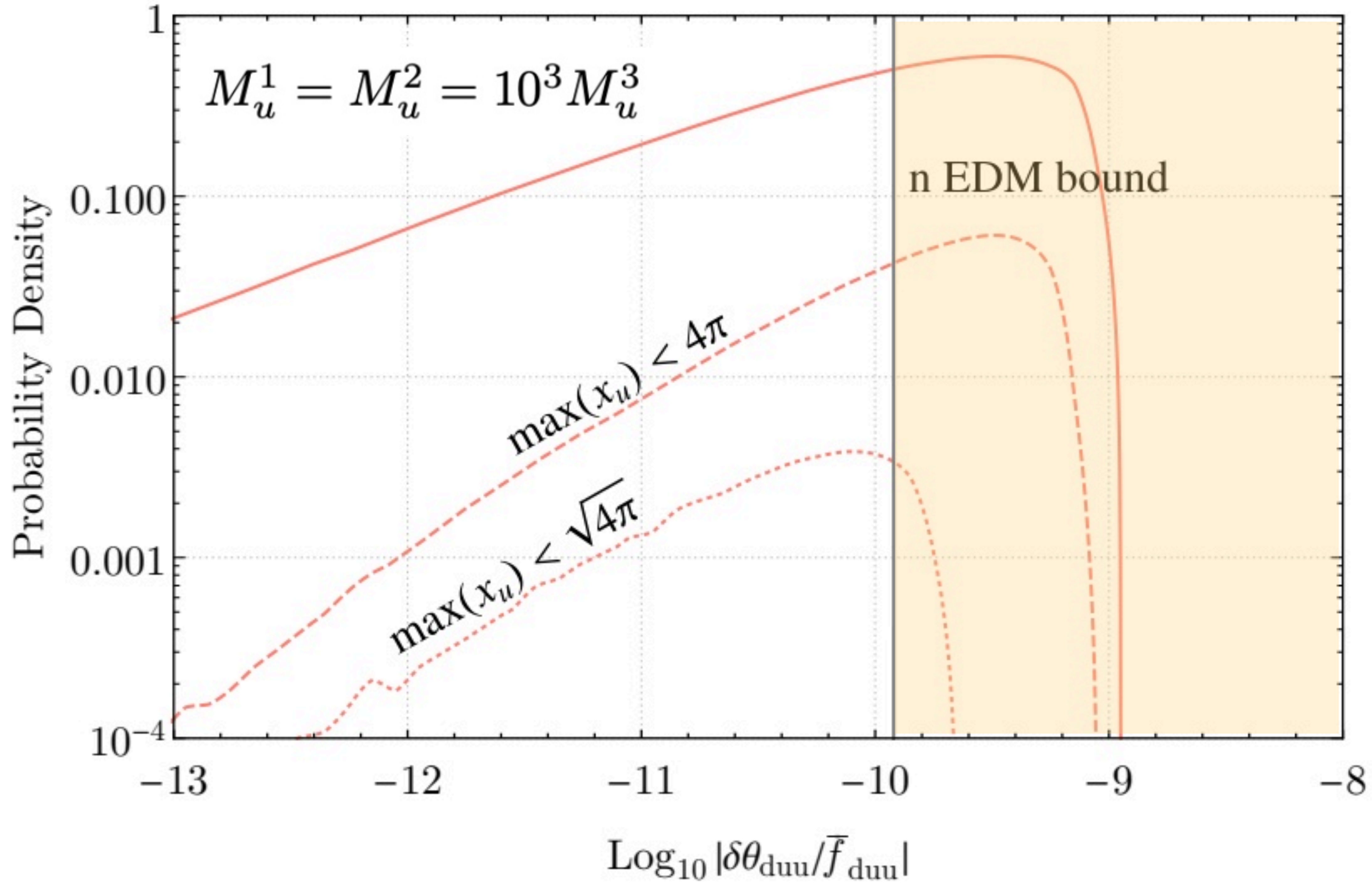
Solutions of strong CP problem

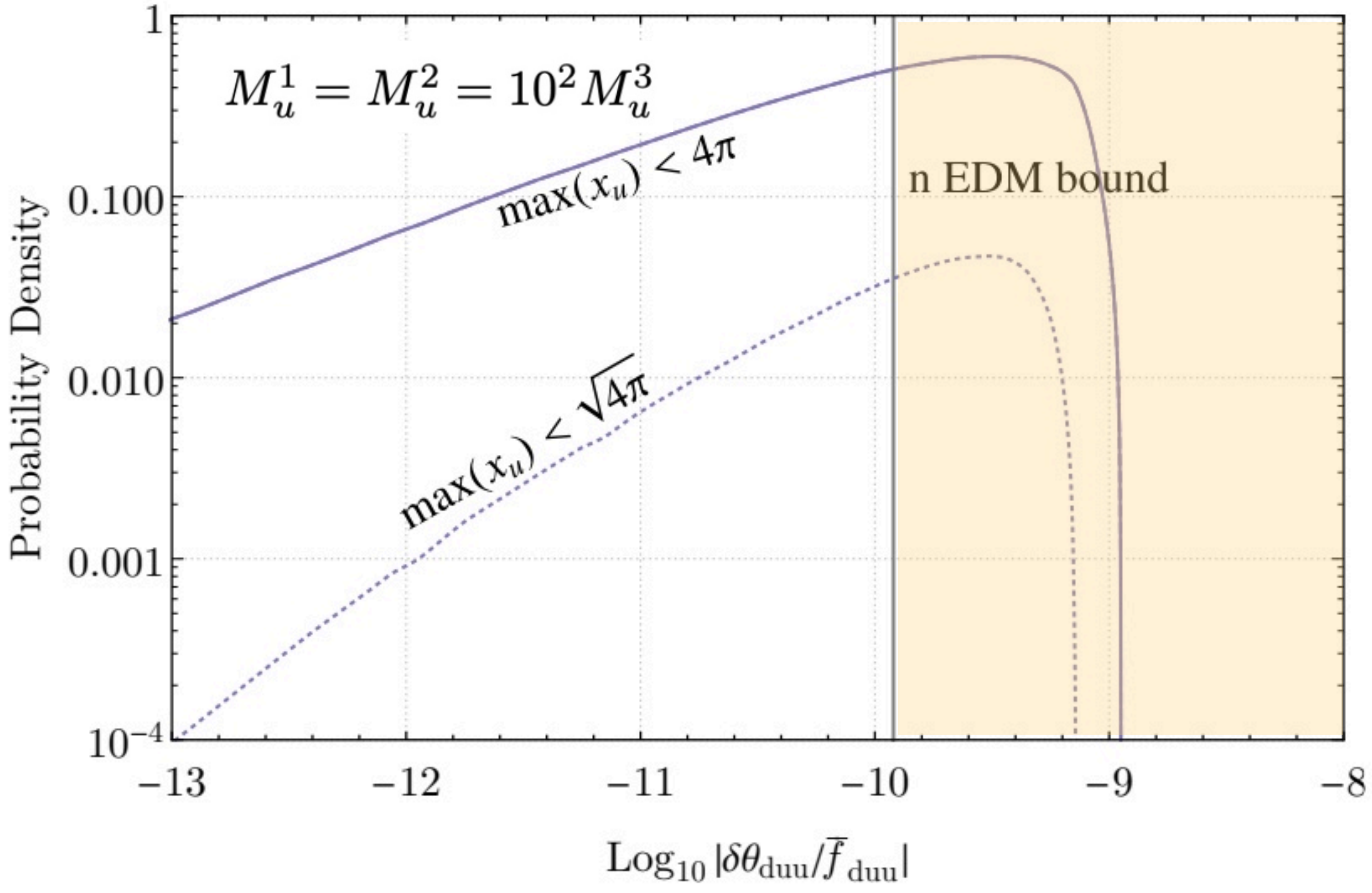
- Axion model: elegant solution, but compatibility with quantum gravity is debatable [axion quality problem] [Kamionkowski, March-Russell, hep-th/9202003, Barr, Seckel, ’92, Holman, et al., hep-ph/9203206]
- Massless up-quark theory: $y_u = 0$ with m_u from nonperturbative effect, but already ruled out by lattice QCD simulation [Alexandrou, et al., 2002.07802]
- **Discrete symmetry:** forbid the bare θ parameter [Spontaneous CP violation, Nelson, ’84, Barr, ’84; P violation Babu, Mohapatra, ’90, Barr, et al., ‘91]
  - Discrete symmetry is softly and explicitly broken. Soft: to produce the SM spectrum and the CKM-phase. Explicit by Planck suppressed: to avoid quality problem and domain wall problem. Radiative θ (from soft breaking) is an interesting topic

3-loop radiative θ in new physics model

- LR対称性模型における、θへの3ループ量子補正と中性子EDMからの制限

[Hisano, TK, Osamura, Yamada, 2301.13405]



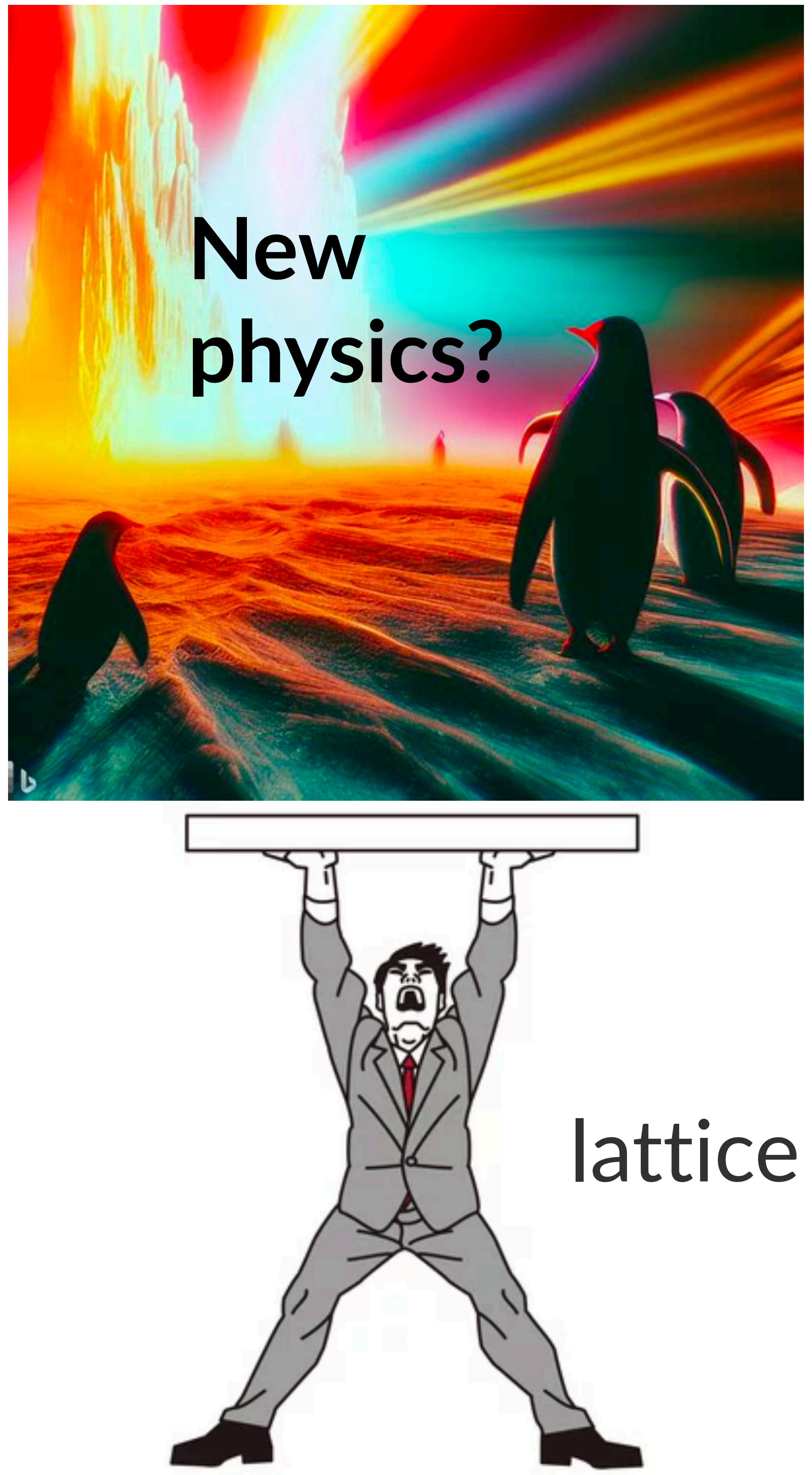


$\approx 50\ \%$ of parameter region would be excluded by nEDM and the perturbative unitarity bounds

まとめ: Take-home messages

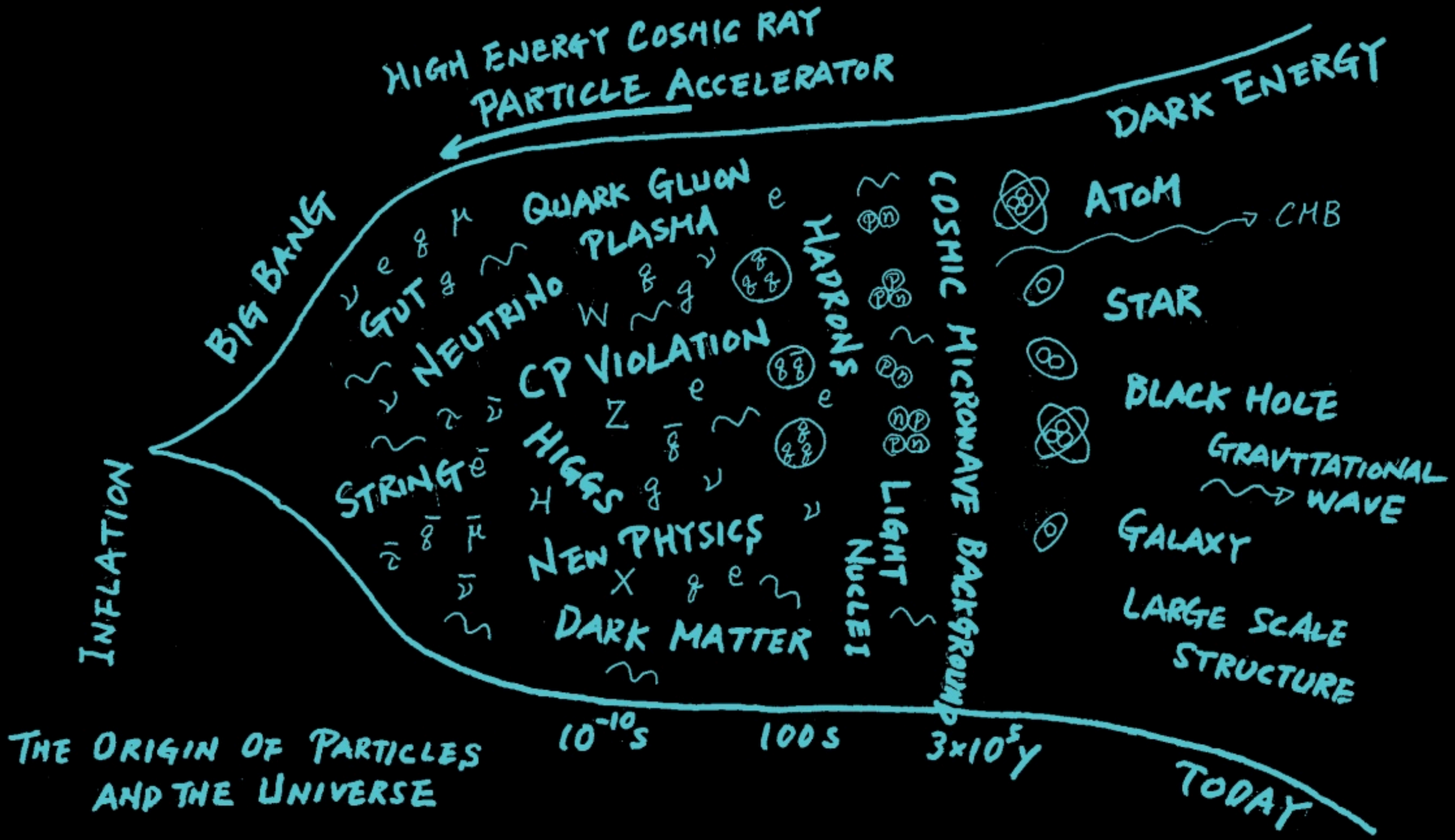
- QCDが与える非摂動な寄与、特にハドロン行列要素は格子QCDで第一原理計算が可能
- 格子QCD計算の数値誤差が年々減少している
- いくつかの事象では、格子QCDが新物理発見の鍵を握っている
  - CKM行列要素決定・ミューオンg-2・中性子EDM

Present and Future



lattice QCD

Backup slides



(c) KMI/Nagoya-U

Wolfenstein four parameters

- Based on the fact $1 \gg sin\theta_{12} \gg sin\theta_{23} \gg sin\theta_{13}$, one can parametrize the CKM matrix by 4-real parameter (with Taylor expansion by λ^n)

$$\sin\theta_{12} \equiv \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}$$

$$\sin\theta_{23} \equiv A\lambda^2 = \frac{|V_{cb}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}$$

$$\sin\theta_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta) = V_{ub}$$

$$(\rho + i\eta) = \frac{\sqrt{1 - A^2\lambda^4}(\bar{\rho} + i\bar{\eta})}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar{\rho} + i\bar{\eta})]}$$

4 fitting parameters (assuming unitarity)

$$\lambda, A, \bar{\rho}, \bar{\eta}$$

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

phase convention independent

Wolfenstein parametrization

- Wolfenstein parametrization at leading order is expressed as

$$V_{\rm CKM} \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

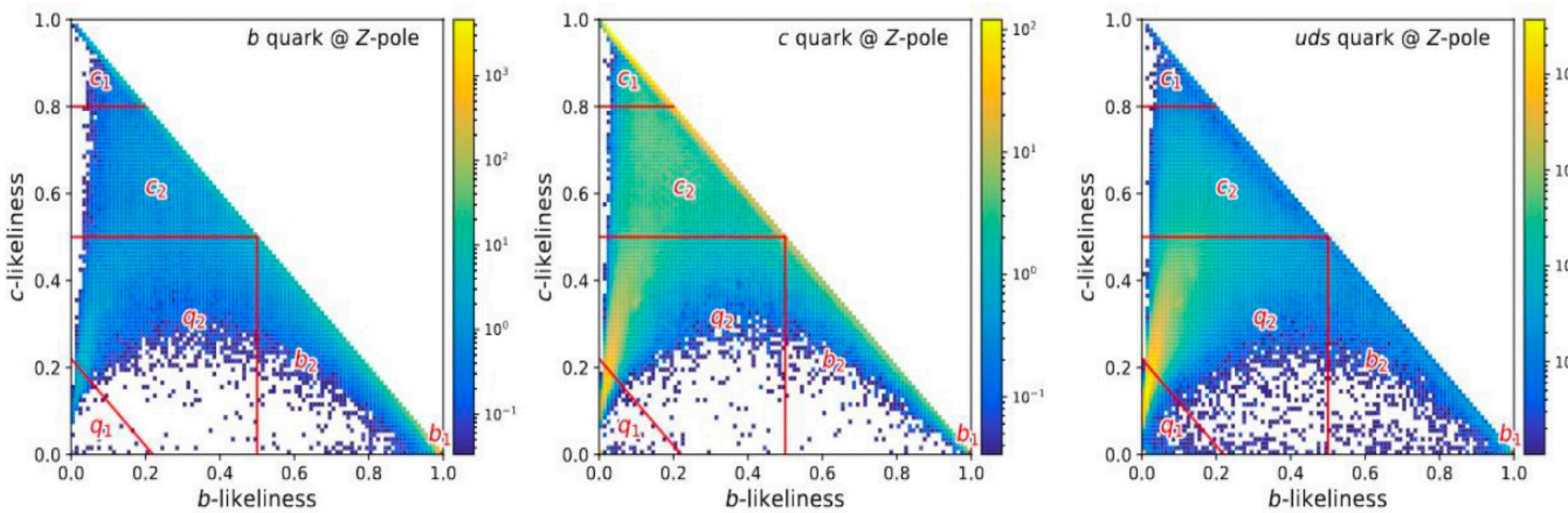
It is unitary up to $O(\lambda^4)$ terms.

Fittings of CPV physics are insufficiency at this order; $V_{ts} \supset -iA\lambda^4\eta, V_{cd} \supset -iA^2\lambda^5\eta,$

so that the phase convention independent $\bar{\rho} + i\bar{\eta}$ parameters must be introduced

$|V_{cb}|$ from W exclusive decay

- CEPC plans to probe $|V_{cb}|$ from $e^+e^- \to W^+W^-, W \to bc, W \to \ell\nu$

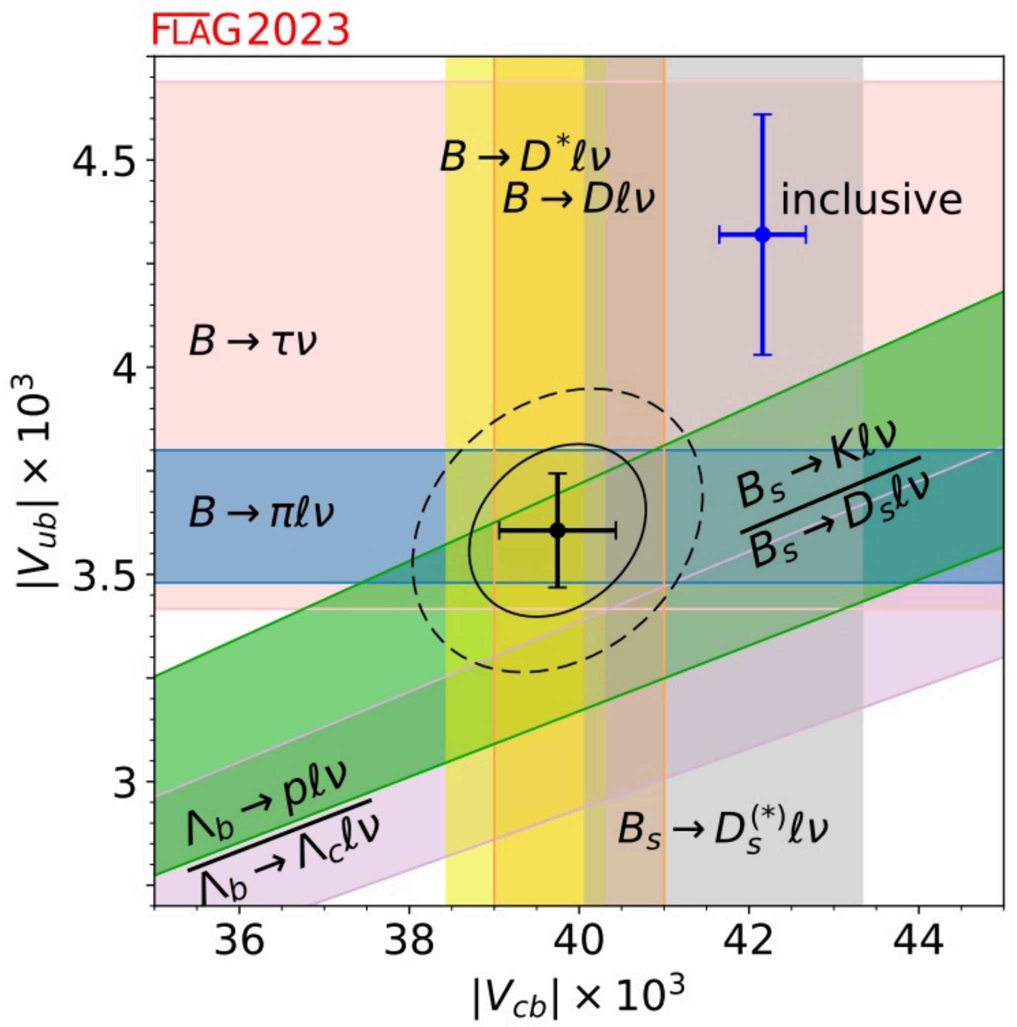


quark \ tag	b_1	b_2	c_1	c_2	q_1	q_2
b	0.47	0.378	0.0197	0.0965	0.00397	0.0315
c	0.00042	0.078	0.298	0.373	0.0682	0.182
uds	0.000104	0.00477	0.00145	0.054	0.538	0.401

[Ruan, et al, 2406.01675; Marzocca, et al, 2405.08880]

$|V_{cb}|$ could be measured to a relative uncertainty of 0.4-1.5% (stat), ~1% (syst) at the CEPC

[FLAG2023]



1.2% inclusive

exclusive fit 1.7%

W inclusive decay

- Inclusive-hadronic decay of W boson, $W \to q\bar{q}'$, is proportional to (in the massless q limit)

$$\propto \underbrace{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2}_{1} + \underbrace{|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2}_{1}$$

= 2 in the CKM unitarity

= $1.9987^{+0.0016}_{-0.0014}$ from flavor

- $W \to q\bar{q}'$ can determine $|V_{cs}|$ and it probes the CKM unitarity test directly

[d'Enterria, Srebre, 1603.06501; CMS, 2201.07861]

CMS Run2 35.9fb-1 result:

$$\mathrm{BR}(W \to q\bar{q}') = (67.46 \pm 0.04_{\mathrm{stat}} \pm 0.28_{\mathrm{syst}})\ \%$$

direct measurement

$$\mathrm{BR}(W \to q\bar{q}') = (67.32 \pm 0.02_{\mathrm{stat}} \pm 0.23_{\mathrm{syst}})\ \%$$

assuming LFU

	$\lvert V_{cs} \rvert$	unitarity test
CMS Run2	0.967 (11)	1.984 (21)
flavor[PDG]	0.975 (6)	1.9987 ($^{+16}_{-14}$)

Light NP: sterile neutrinos?

- New right-handed gauge-singlet fermions N_I $(I = 1,2,\ldots)$ are introduced

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N}_I \partial\!\!\!/ N_I - (\overline{L}_\ell \tilde{H}) y^{\ell I} N_I - \frac{1}{2} M_I \overline{N}_I^c N_I + \text{ h.c.}$$

- The mass matrix and mixings

$$\mathcal{L}_{\rm mass} = -\frac{1}{2} \begin{pmatrix} \overline{\nu}_\ell & \overline{N}_I^c \end{pmatrix} \begin{pmatrix} 0 & M_D \\ M_D^T & M_I \end{pmatrix} \begin{pmatrix} \nu_\ell^c \\ N_I \end{pmatrix} + \text{h.c.} \qquad M_D = v y^{\ell I}$$

mass eigenstates

$$m_{\rm light} = \mathcal{O}\left(\frac{y^2 v^2}{M}\right), \quad m_{\rm heavy} = \mathcal{O}(M)$$

active-sterile mixing matrix $\quad U_{\ell I} = \dfrac{v y^{\ell I}}{M_I}$

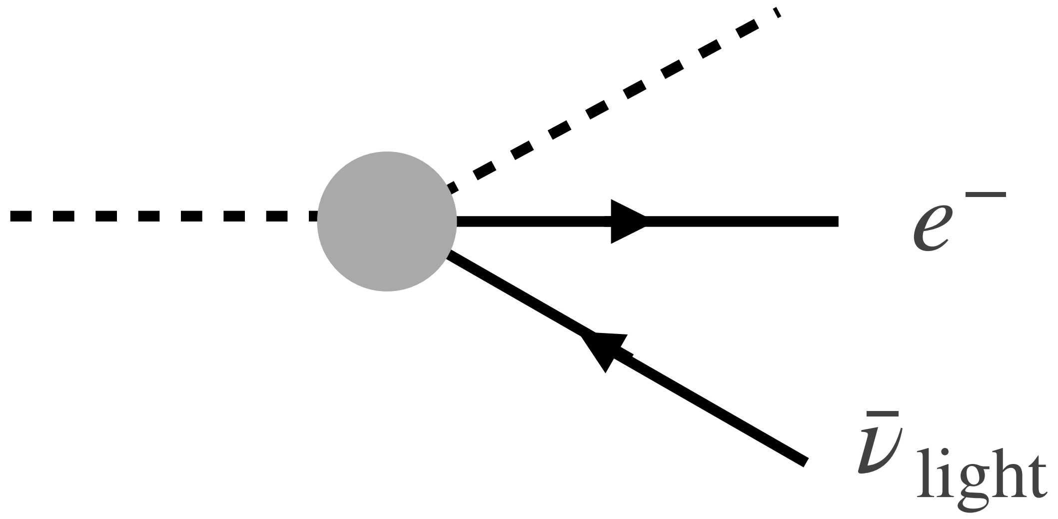
massive SM neutrinos

Sterile neutrinos

Sterile neutrino contributions

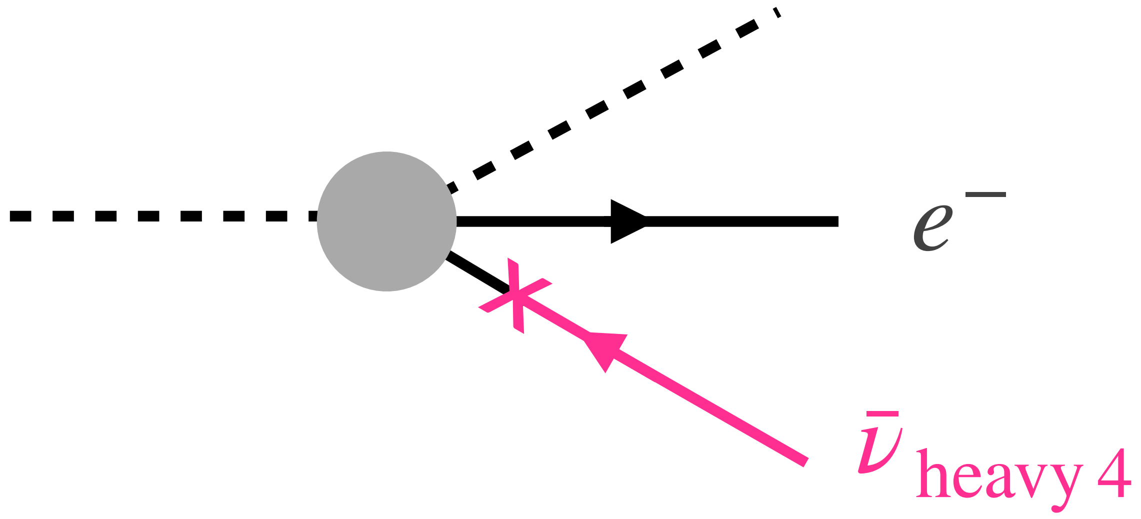
- **Two contributions** to the leptonic and semi-leptonic decays

1. modifies active neutrino coupling



$\propto \cos\theta_e$

2. decay into sterile neutrino if kinematically possible



$\propto \sin\theta_e^{(4)}$ with phase space suppression

- When the sterile neutrino masses are much smaller than the decay Q-value $(M_N \ll Q)$, **the total contribution from 1+2 is canceled**

$$|\mathcal{M}|^2 = |\mathcal{M}_{\text{SM}}|^2 \cos^2\theta_e + |\mathcal{M}_{\text{SM}}|^2 \sin^2\theta_e \times f(M_N, Q)$$

[Isakov, Strikman, ’86; Deutxh, Lebrun, Prieels, '90]

$$\simeq |\mathcal{M}_{\text{SM}}|^2 (\cos^2\theta_e + \sin^2\theta_e) = |\mathcal{M}_{\text{SM}}|^2$$

sterile-neutrino contributions are suppressed when $M_N \ll Q$

Super-allowed ($0^+ \to 0^+$) nuclear β decays

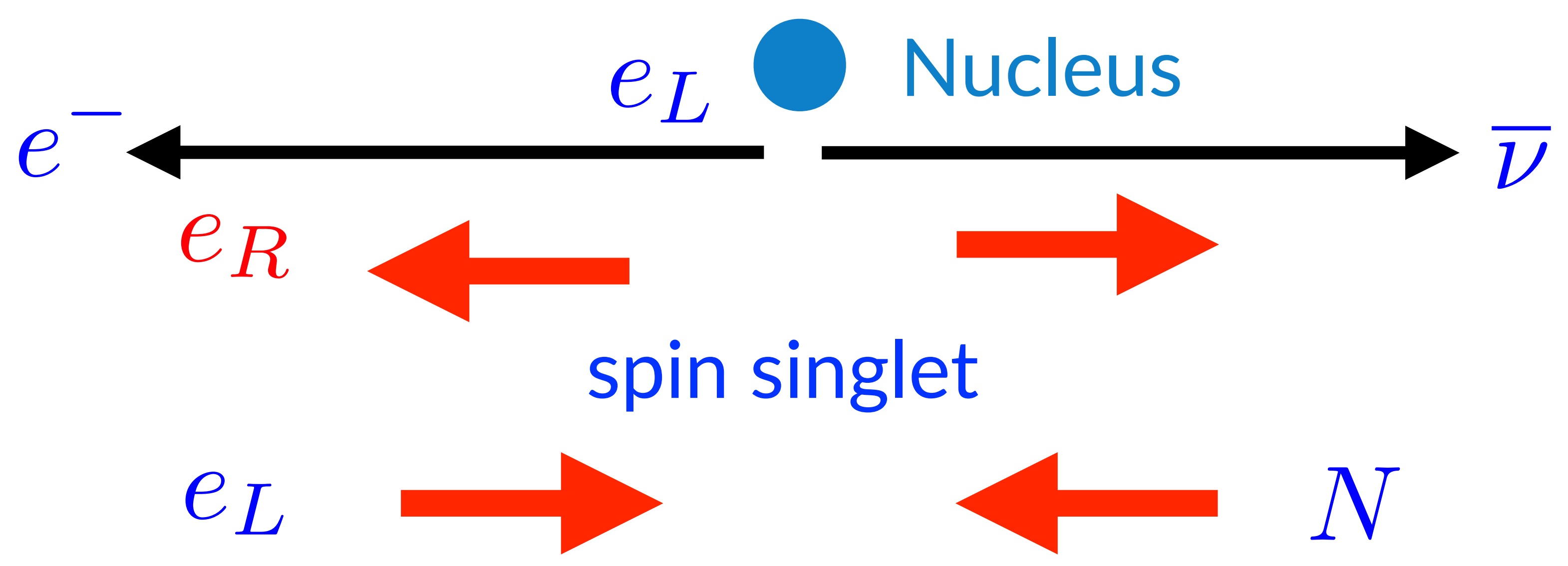
- $|V_{ud}|$ is determined by the global fit of the super-allowed ($0^+ \to 0^+$) nuclear β-decays

$$J^P = 0^+ \to 0^+ \quad \text{with } \beta^+ \text{ decay } (p^+ \to n + e^+ \nu_e)$$

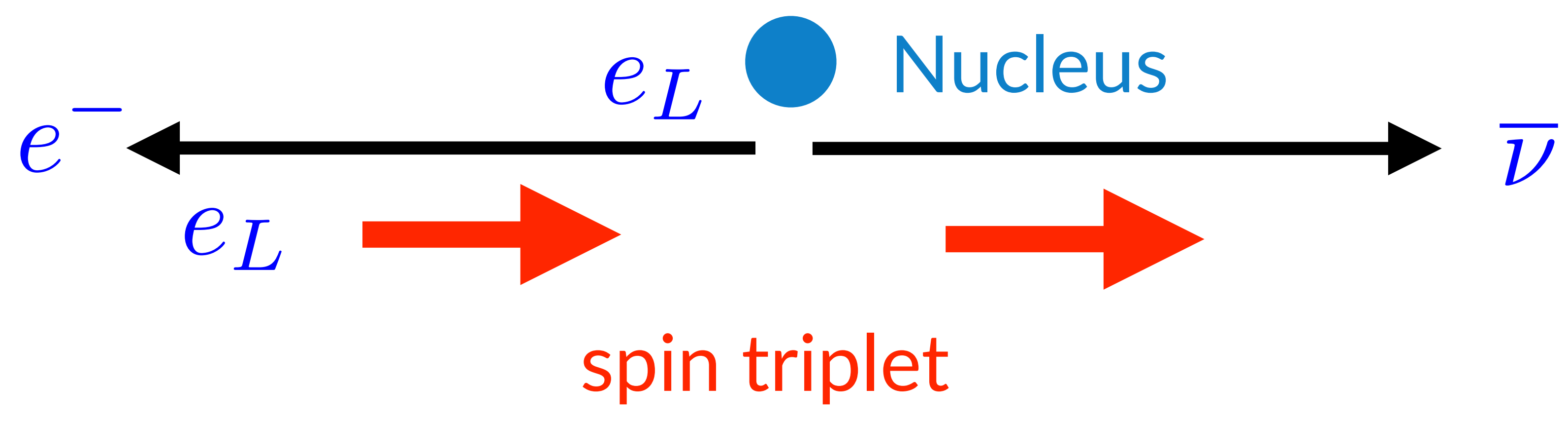
J is total nuclear angular momentum, $P = (-1)^L$ = parity and L is orbital angular

positron-neutrino pair must be spin singlet

Fermi decay (vector current)



Gamow-Teller decay (axial-vector current)



Forbidden in the super-allowed decays

Advantages
1. Theoretically clean and nucleus independent
2. Precisely measurable in experiments

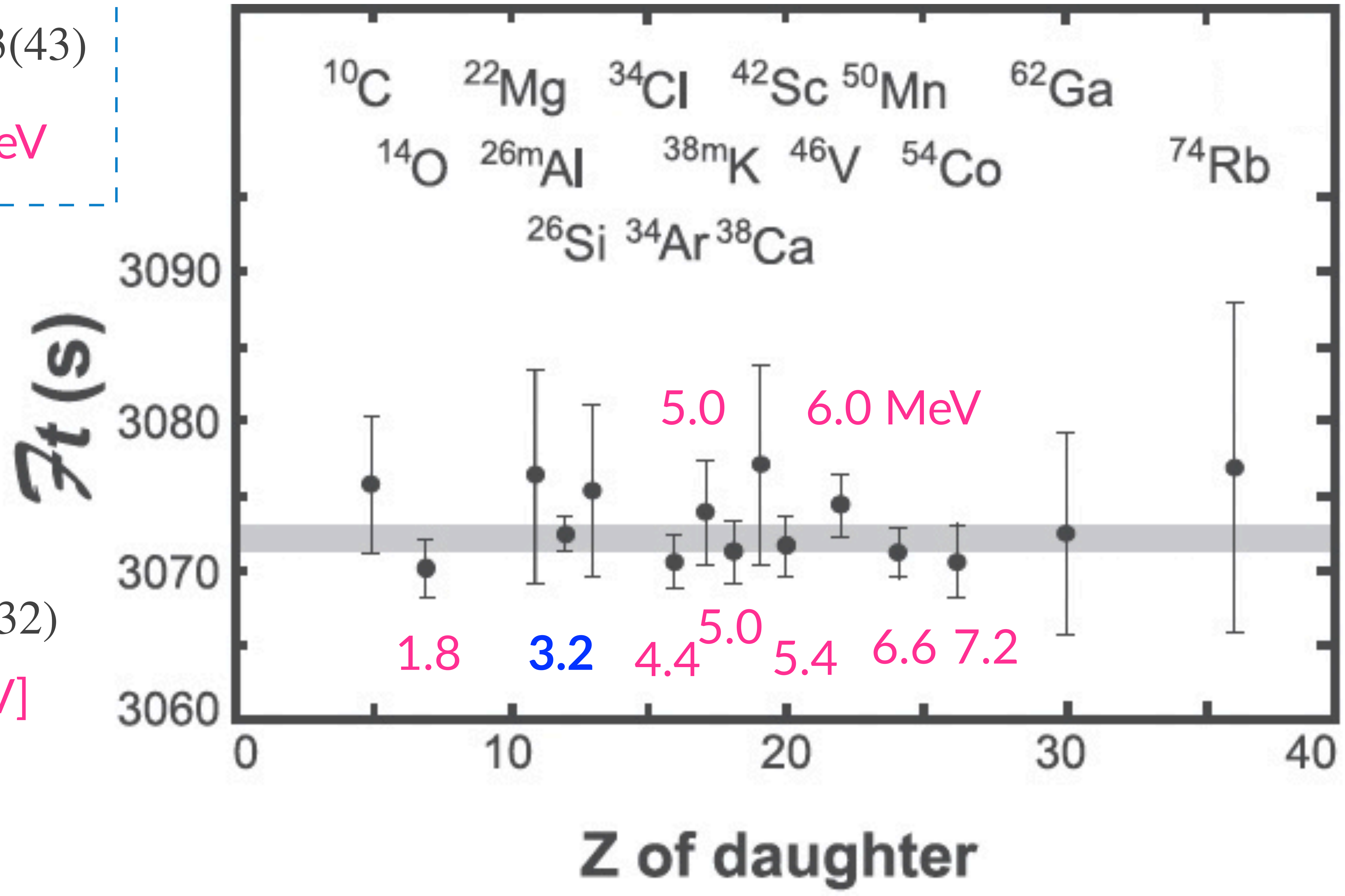
Super-allowed β-decay Q values

[Cirigliano et al, 2208.11707]

$$V_{ud}^{n;\,\mathrm{bottle}} = 0.97413(43)$$

Q = 0.78 MeV

[Hardy, Towner, '20]



There are 15 super-allowed β-decay data

super-allowed

$$V_{ud}^{0^+\to 0^+} = 0.97367(32)$$

Q values [MeV]

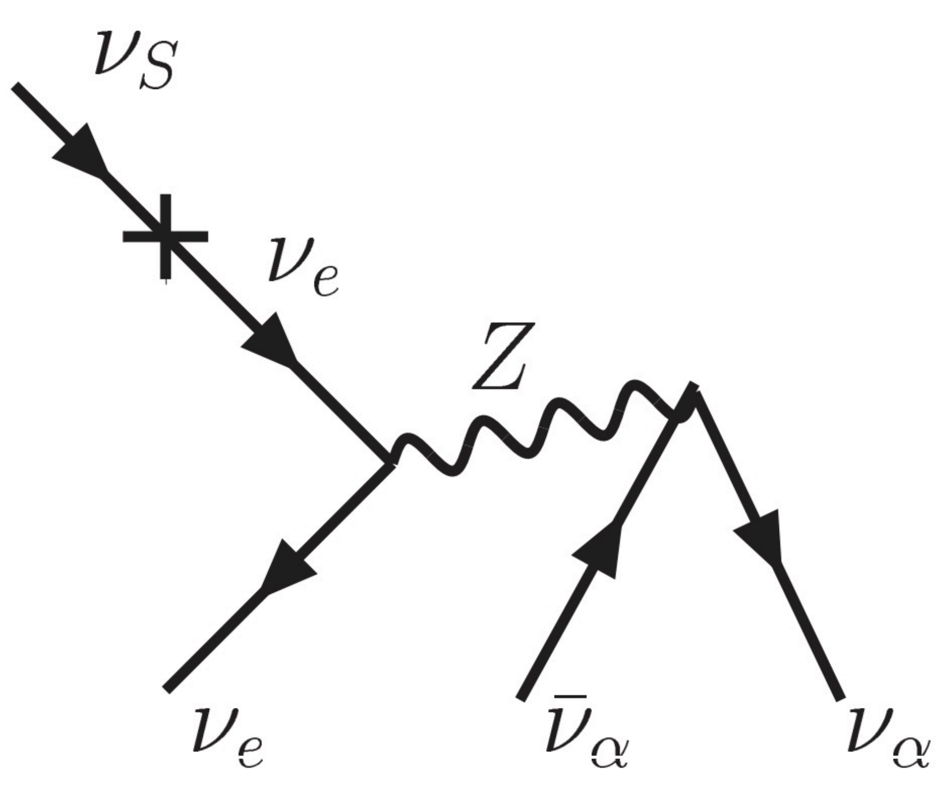
$$Q = E_\nu^{\max}$$

$|V_{ud}|$ is predominantly determined by data of Al → Mg transition (Q=3.2 MeV)

~3MeV sterile neutrino provides a big impact on the CAA tension

Sterile neutrino lifetime

- The O(MeV) sterile neutrino lifetime $(N \to \nu_e \bar{\nu}_\ell \nu_\ell,\ \nu_e e^+ e^-)$ [e.g. 1202.2841, 1504.04855]



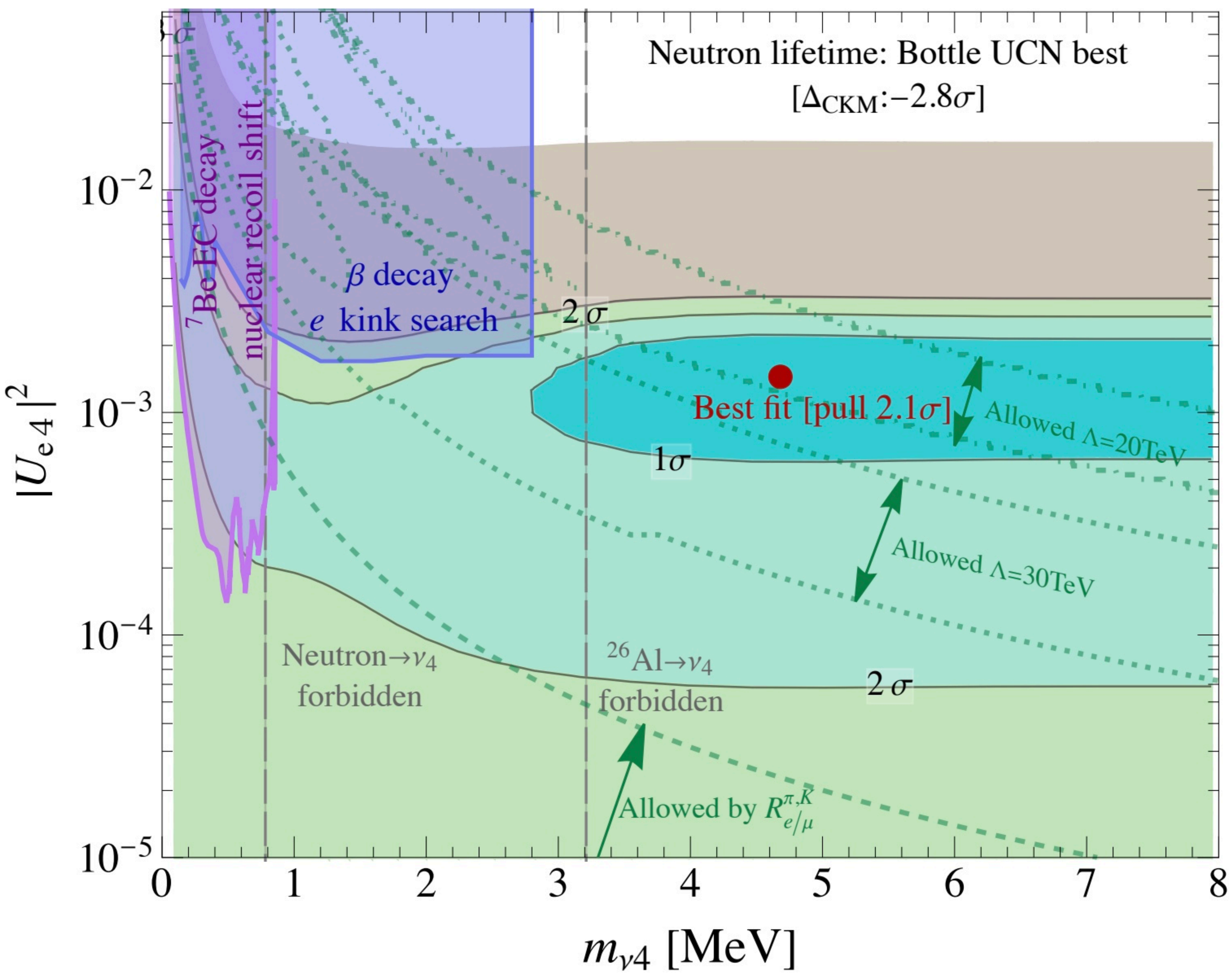
$$\tau_N \sim 300 \times \left(\frac{M_I}{2\mathrm{MeV}}\right)^{-5} \left|U_{eN}\right|^{-2} \text{ sec} \gg \text{ BBN time [100-1000 sec]}$$

Energy injection from such a long-lived sterile neutrino modifies the light nuclei abundance after the BBN;
The MeV sterile neutrino is excluded by the primordial abundance of ^{4}He (Y_p measurement)

- We assume that the O(MeV) sterile neutrino can promptly decay into dark sector to avoid the BBN bound
$N \to \nu_e \phi^* \to \nu_e \gamma\gamma$; we checked that this decay channel is consistent with cosmology

Sterile neutrino solution for CAA

[TK, Tobioka, 2308.13003]



MeV sterile neutrino provides good effects on $|V_{ud}|$ from super-allowed β decays, and no impacts on the other meson decays

But, viable model is challenging;

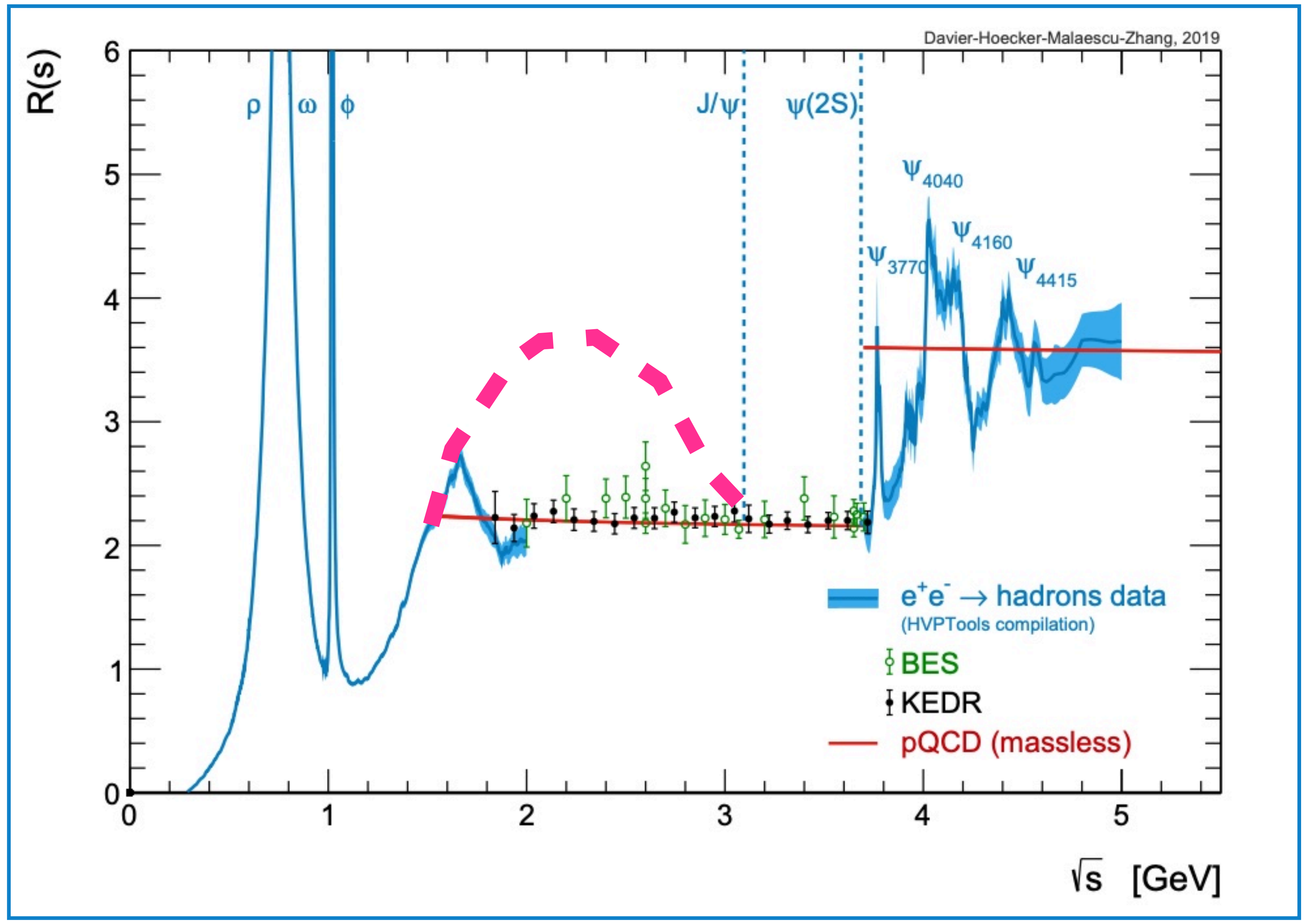
1. To avoid cosmological bounds, mechanism for the shorter lifetime is needed
2. To avoid 0νββ bound, the inverse seesaw model is needed
3. To avoid $\pi^+ \to e^+ N$ bound, additional dim-6 interaction is needed

Near future of $a_\mu(\mathrm{Th})$

[see details, Statement of the Muon g-2 Theory Initiative]

- Long-distance contribution to HVP (BMW) has not been cross-checked by other lattice group
  - The intermediate window observables (⅓ of the total HVP) have been cross-checked well
- CMD-2, CMD-3 and SND collaborations are using the same facility, but only CMD-3 provides the different result of the $\pi^+\pi^-$ cross section
  - New analysis of **BaBar** (in 2024), update analysis of **SND**, update and new analysis of **BES III**, update and new blind analysis of **KLOE**, and **Belle II** result (in 2025) of the $\pi^+\pi^-$ cross sections will be reported in near future
- MUonE (space-like HVP) final-goal result will be announced in LHC Run 4 (~2030)

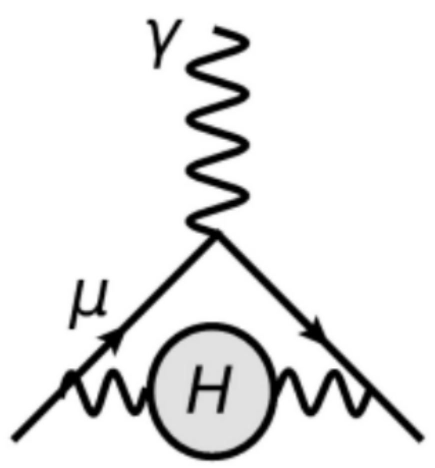
$$\sigma(e^+e^- \to \text{ hadron inclusive})(s)$$
$$= \sum_{\text{exc.}} \sigma(e^+e^- \to \text{ hadron exclusive})(s)$$



$$\sigma(e^+e^- \to \text{ hadron inclusive})(s)$$
$$= \sum_{\text{exc.}} \sigma(e^+e^- \to \text{ hadron exclusive})(s)$$
$$+ \, \sigma(e^+e^- \to \text{unknown QCD})$$

!?

Unknown QCD in HVP (1/2)



[Keshavarzi, Marciano, Passera, Sirlin, 2006.12666]

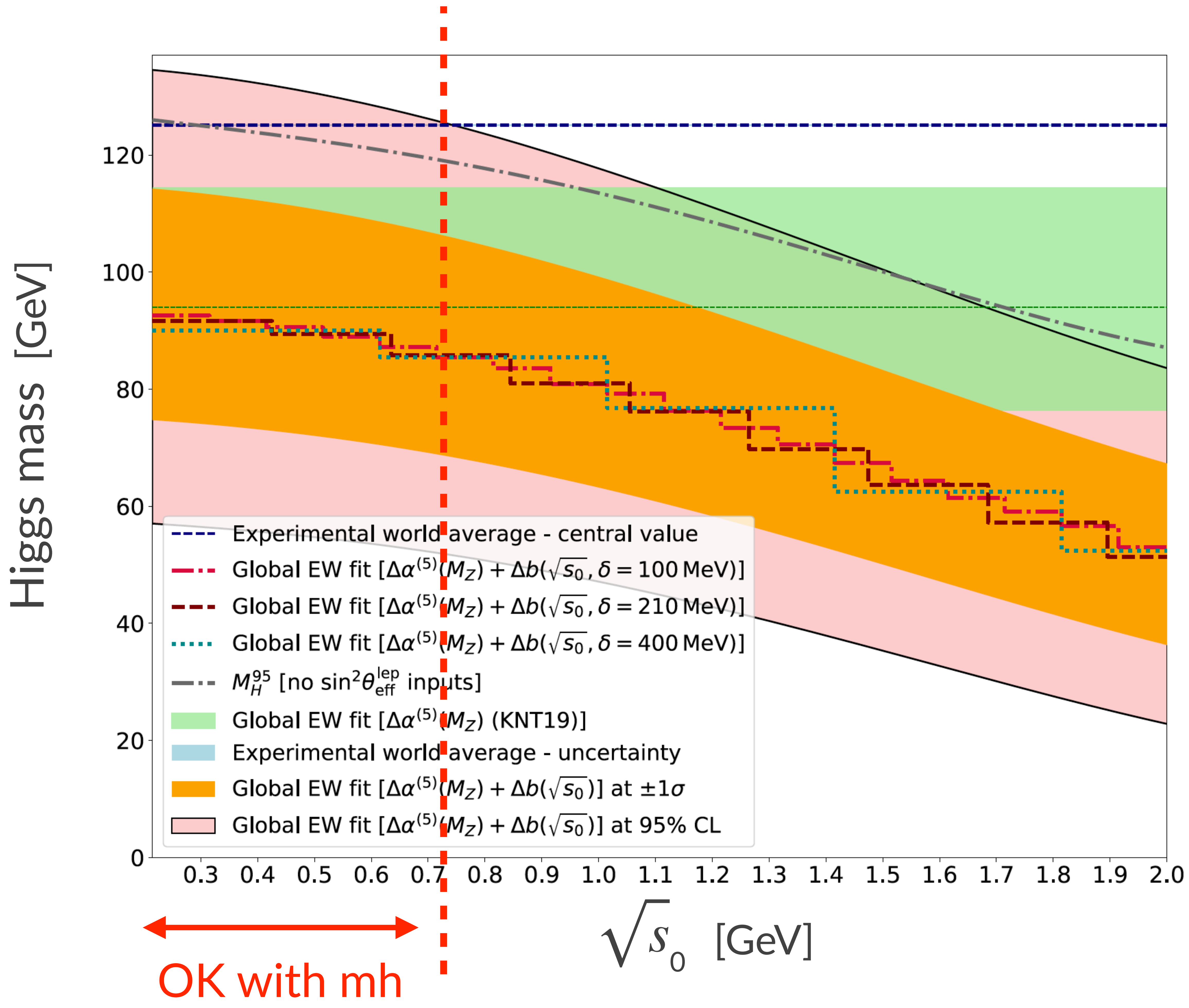
Is there unknown QCD contribution to HVP?
[Today's topic]

↓

This possibility is severely constrained by the electroweak (EW) fit (right figure)

↓

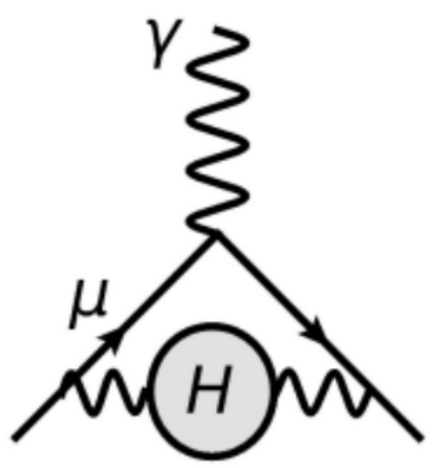
EW fit could be no problem only when the low energy region of $e^+e^- \to \mathrm{hadrons}$ ($\sqrt{s} \lesssim 0.7\,\mathrm{GeV}$) are modified by unknown QCD.



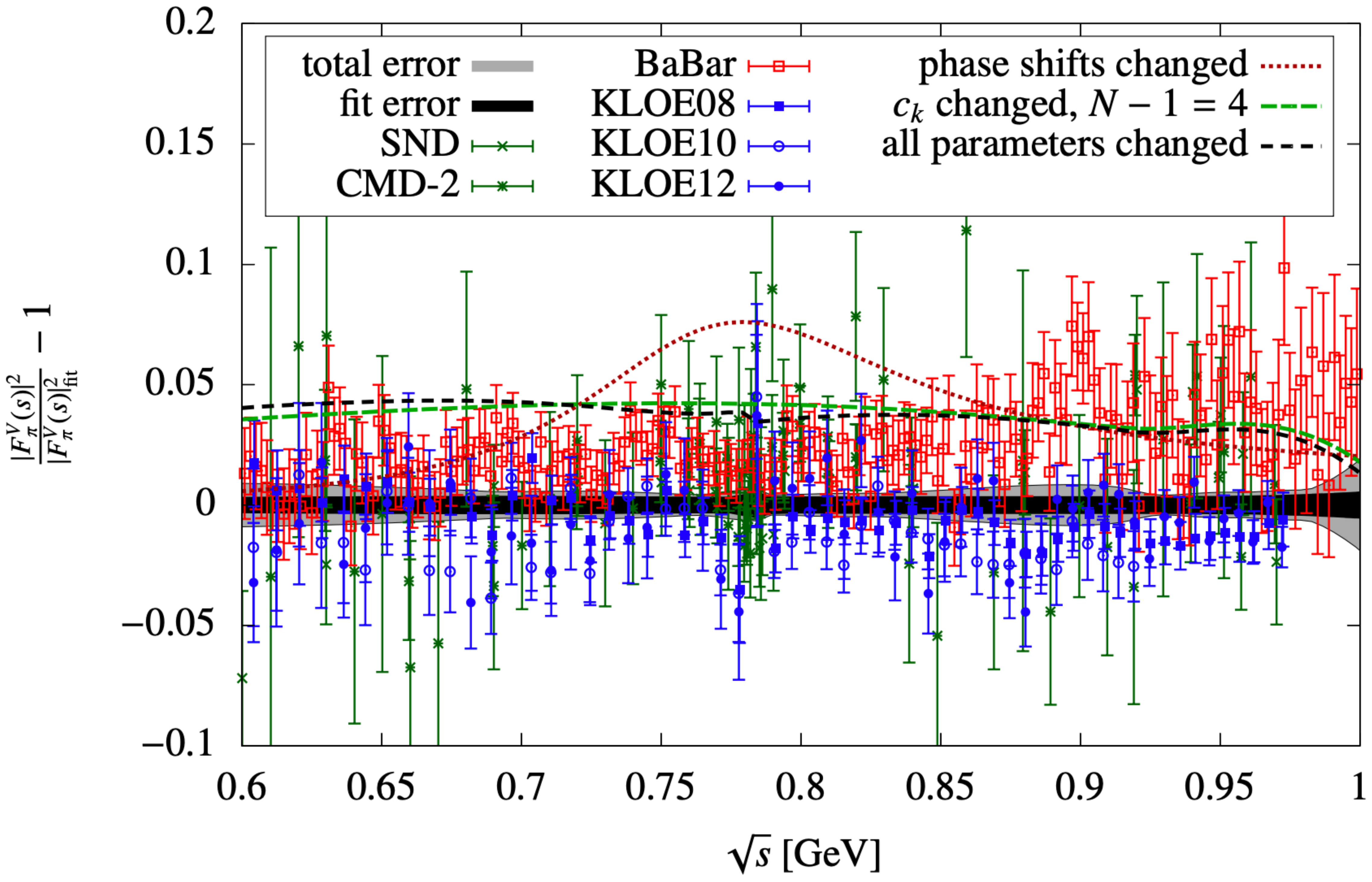
125GeV Higgs

Here:
4σ muon g-2 anomaly is **supposed to be explained by unknown QCD contribution to the HVP** at $\sqrt{s_0}$ GeV

Unknown QCD in HVP (2/2)



- Focus on $e^+e^- \to \text{hadrons}\ \sqrt{s} \lesssim 0.7\,\text{GeV}$ [Colangelo, Hoferichter, Stoffer, 2010.07943]



data fit [KNT]

When one considers that the lattice value can be explained by the unknown QCD effects ($\sqrt{s} \lesssim 0.7\,\text{GeV}$),

then there is additional tension:

8% change of $e^+e^- \to \rho$ resonance (·········),

or

4% change of $e^+e^- \to 2\pi$ data (- - - - -),

while data fit has **only 1% error**

[Keshavarzi, Nomura, Teubner, 1911.00367]

Updated data ($e^+e^- \to \text{hadrons}$) will be provided by **Belle II experiment.**

Belle II

BLR + bino-stau coannihilation

Benchmark point

Bino-stau coannihilation
with correct $\Omega_{\rm DM}$
+ universal slepton mass

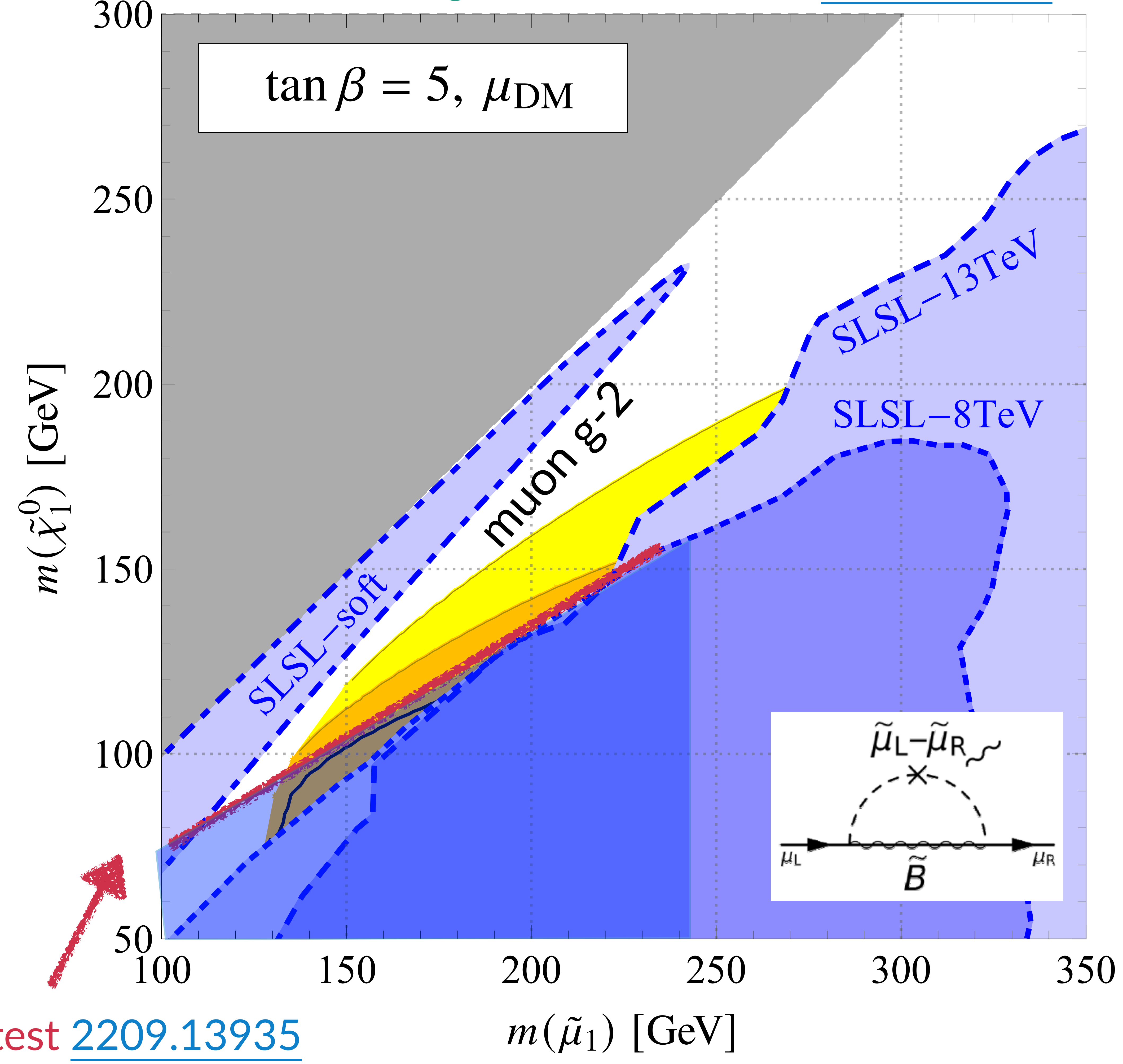
Large μ with small $\tan\beta$ is favored in this study
(BHR gives negative contribution)

strong bound from:

$$\tilde{\ell}\tilde{\ell}^* \to (\ell\tilde{\chi}_1^0)\ (\bar{\ell}\tilde{\chi}_1^0)$$

stau mass < 200 GeV
→ good target for **ILC500**

[Endo, Hamaguchi, Iwamoto, TK, 2104.03217]



ATLAS the latest 2209.13935

Benchmark points

	BLR1	BLR3
M_1	100	150
$m_{\rm L} = m_{\rm R}$	150	200
$\tan\beta$	5	5
μ	1323	1922
$m_{\tilde{\mu}_1}$	154	202
$m_{\tilde{\mu}_2}$	159	207
$m_{\tilde{\tau}_1}$	113	159
$m_{\tilde{\tau}_2}$	190	242
$m_{\tilde{\nu}_{\mu,\tau}}$	137	190
$m_{\tilde{\chi}_1^0}$	99	150
$m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_1^\pm}$	1323–1324	1922–1923
$a_\mu^{\rm SUSY} \times 10^{10}$	27	17
$\Omega_{\rm DM}h^2$	0.120	0.120
$\sigma_p^{\rm SI} \times 10^{47}$ [cm^2]	1.7	0.8
$\mu_{\gamma\gamma}$	1.01	1.01

XENONnT (DM direct detection) can probe this scenario

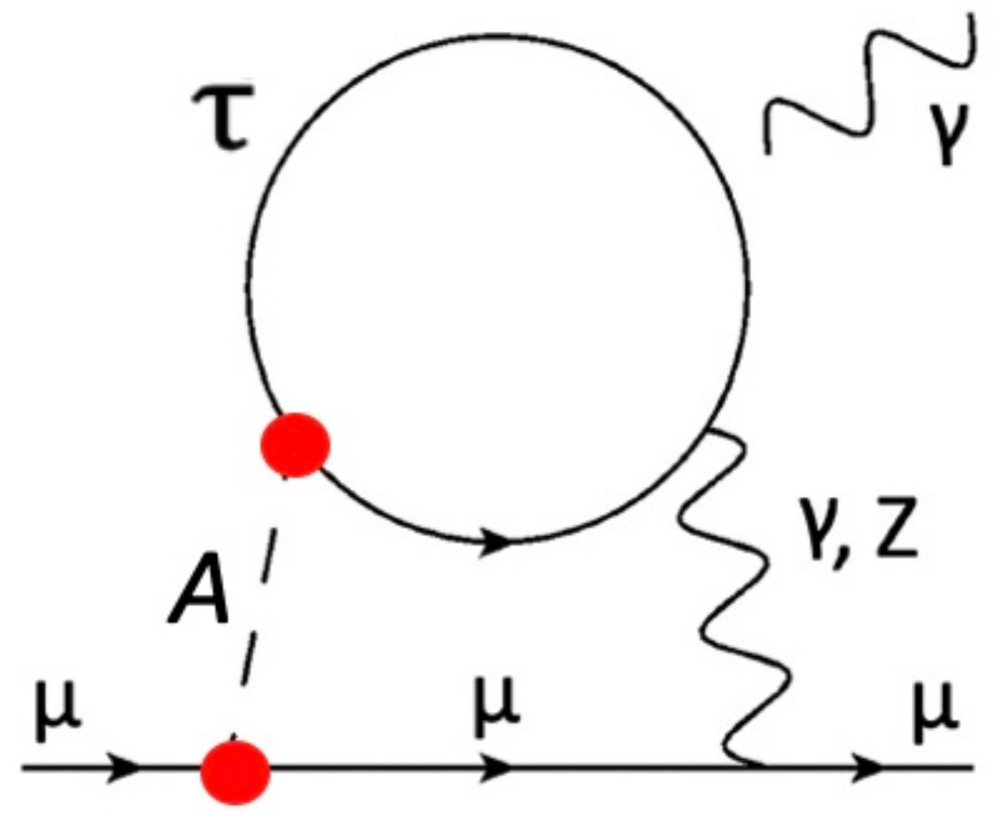
The simplest explanation in 2HDM was excluded

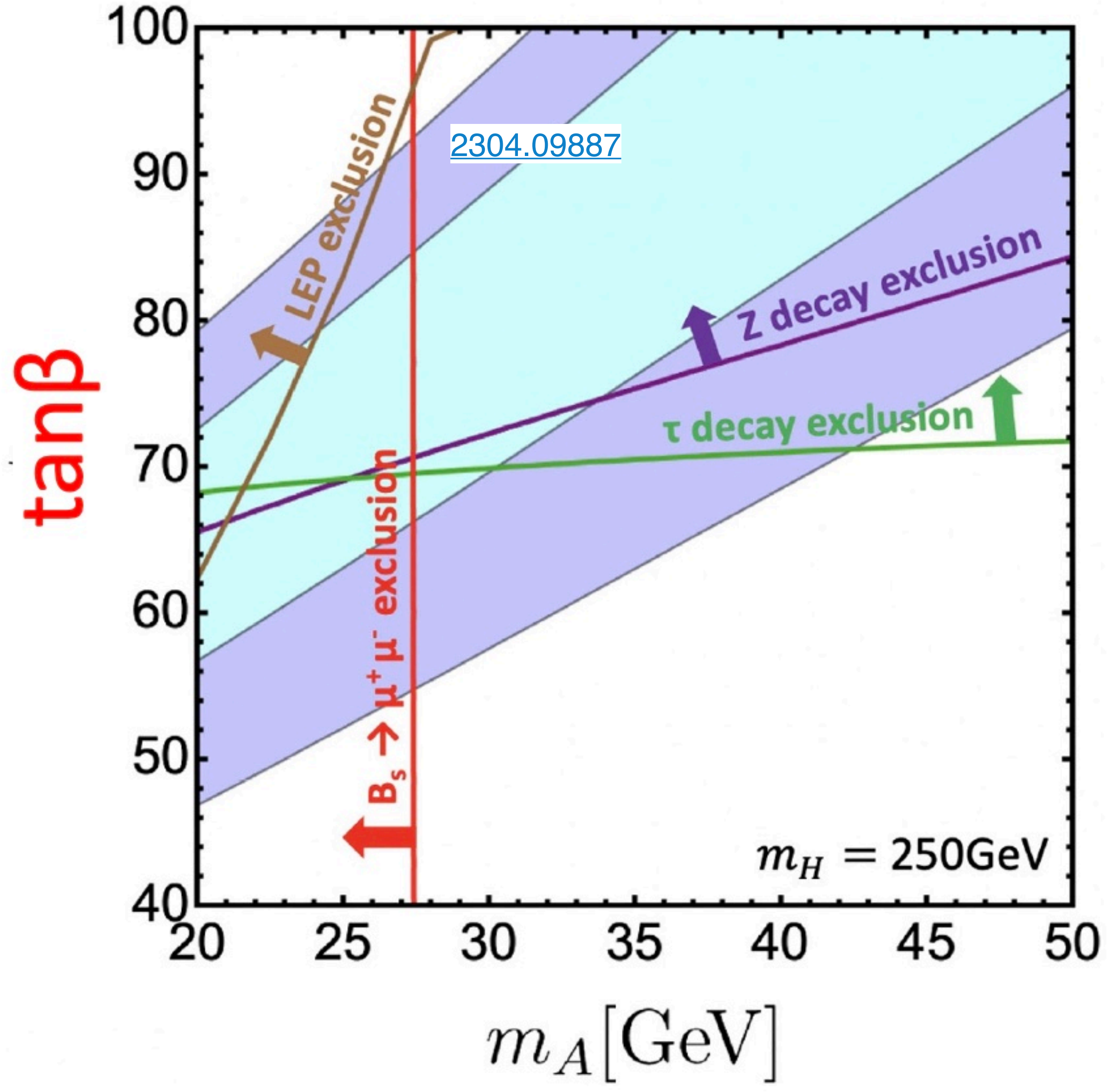
- There was a room that muon g-2 anomaly can be explained by lepto-philic (type-X) 2HDM

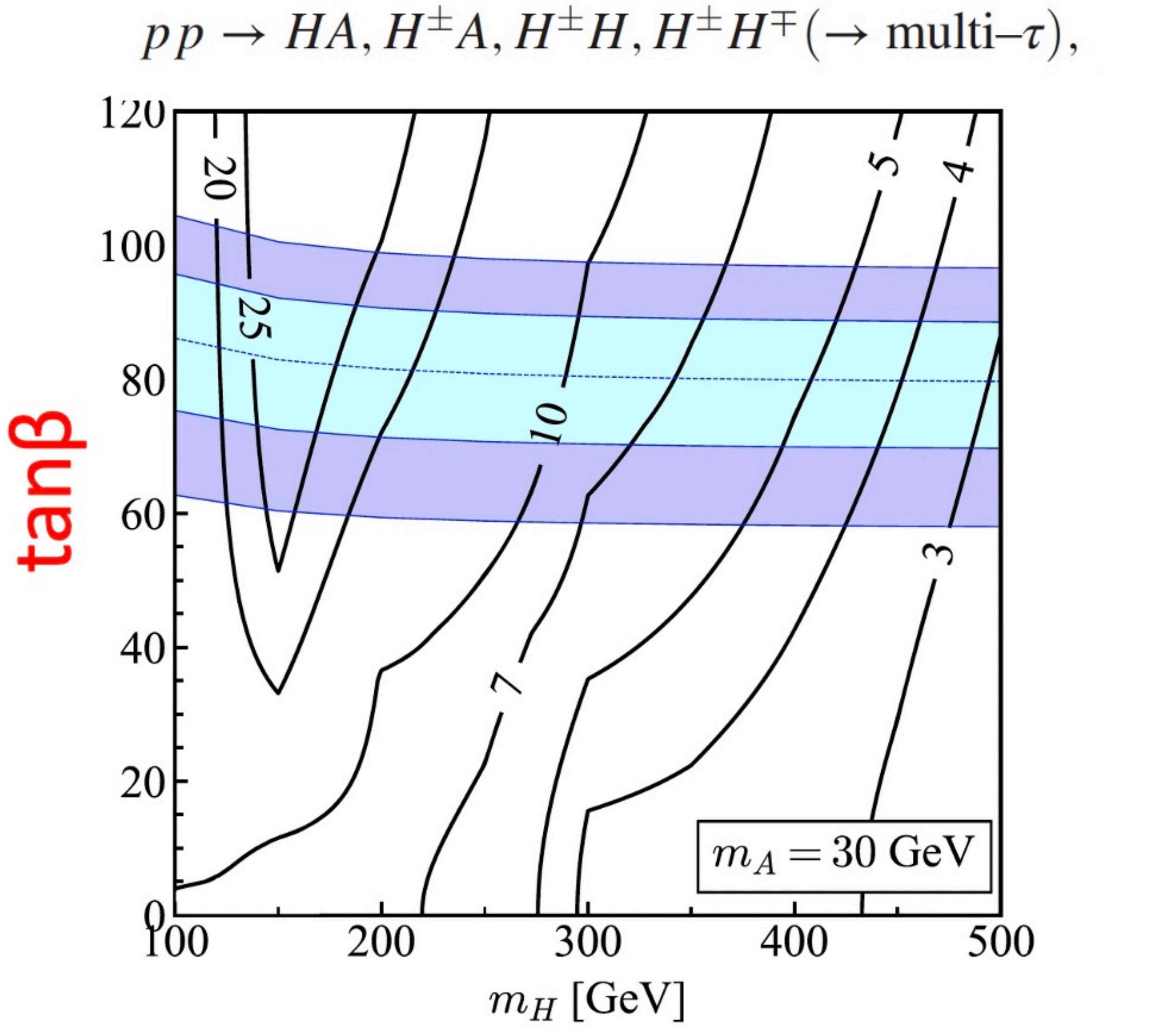
[Takeuchi, Iguro, TK, Lang, 2304.09887]

- It is found that such a 2

$\tan\beta$







larger than 1 is expected to be excluded in Run2 data

標準模型における問題点

- 量子色力学（強い力）の数学的理理解
- 暗黒物質・暗黒エネルギーの解明
- 大統一理論の謎、階層性問題の謎
- ニュートリノ質量機構
- フレーバーと世代の起源の解明
- インフレーション宇宙の解明
- 宇宙のバリオン数生成の謎
- 真空・時空構造の理解
- 量子重力・超弦理論の解明

これらの謎は

「標準模型を超える物理」

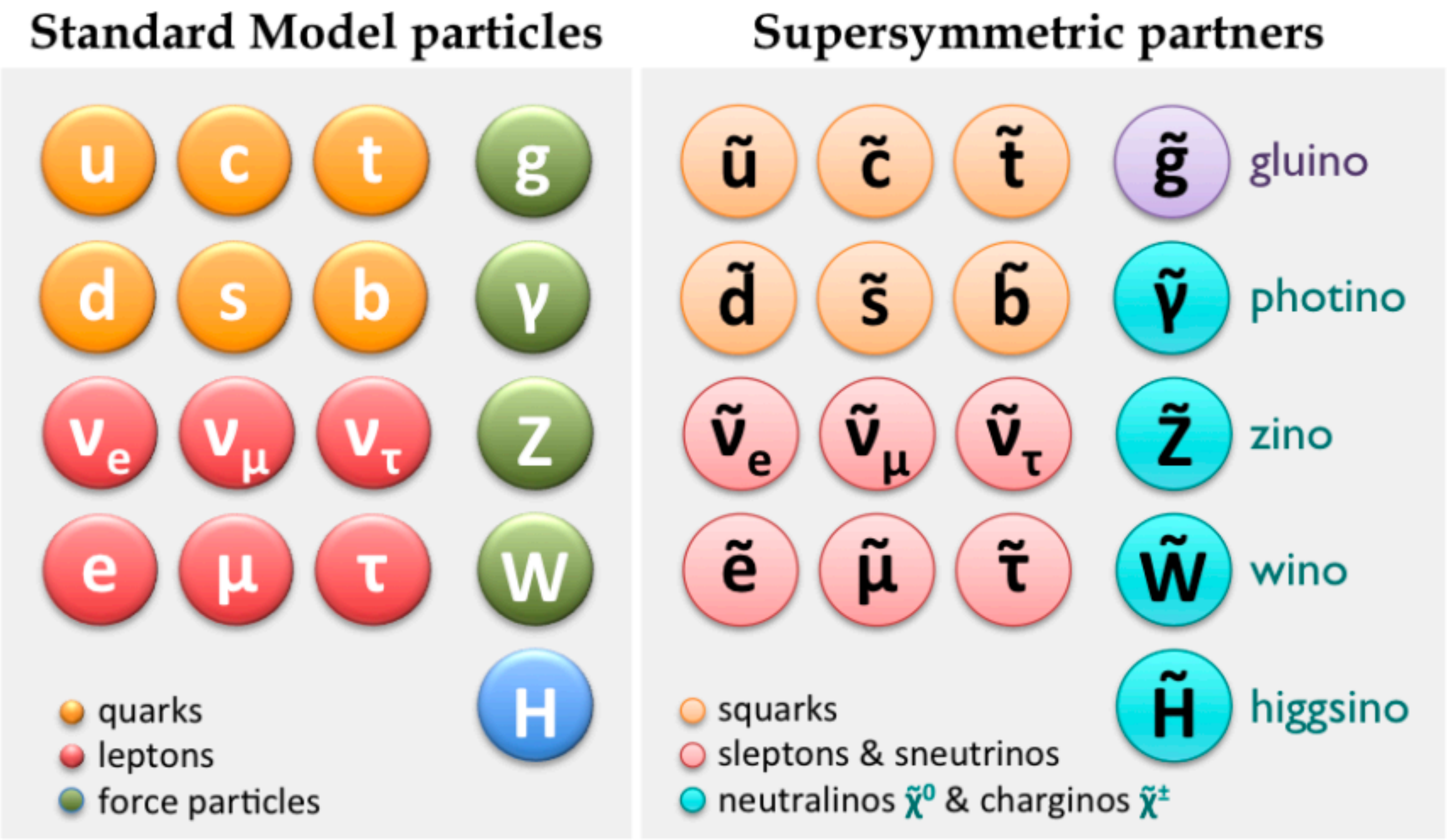
が確かに存在すること
を示唆している

超対称性理論

- 最も人気が「あった」標準模型を超える物理が**超対称性理論**
- スピン間が1/2だけ異なる超対称性粒子を導入する（素粒子の種類は倍）

標準模型粒子は「18」

(偶数個のヒッグス粒子を予言)



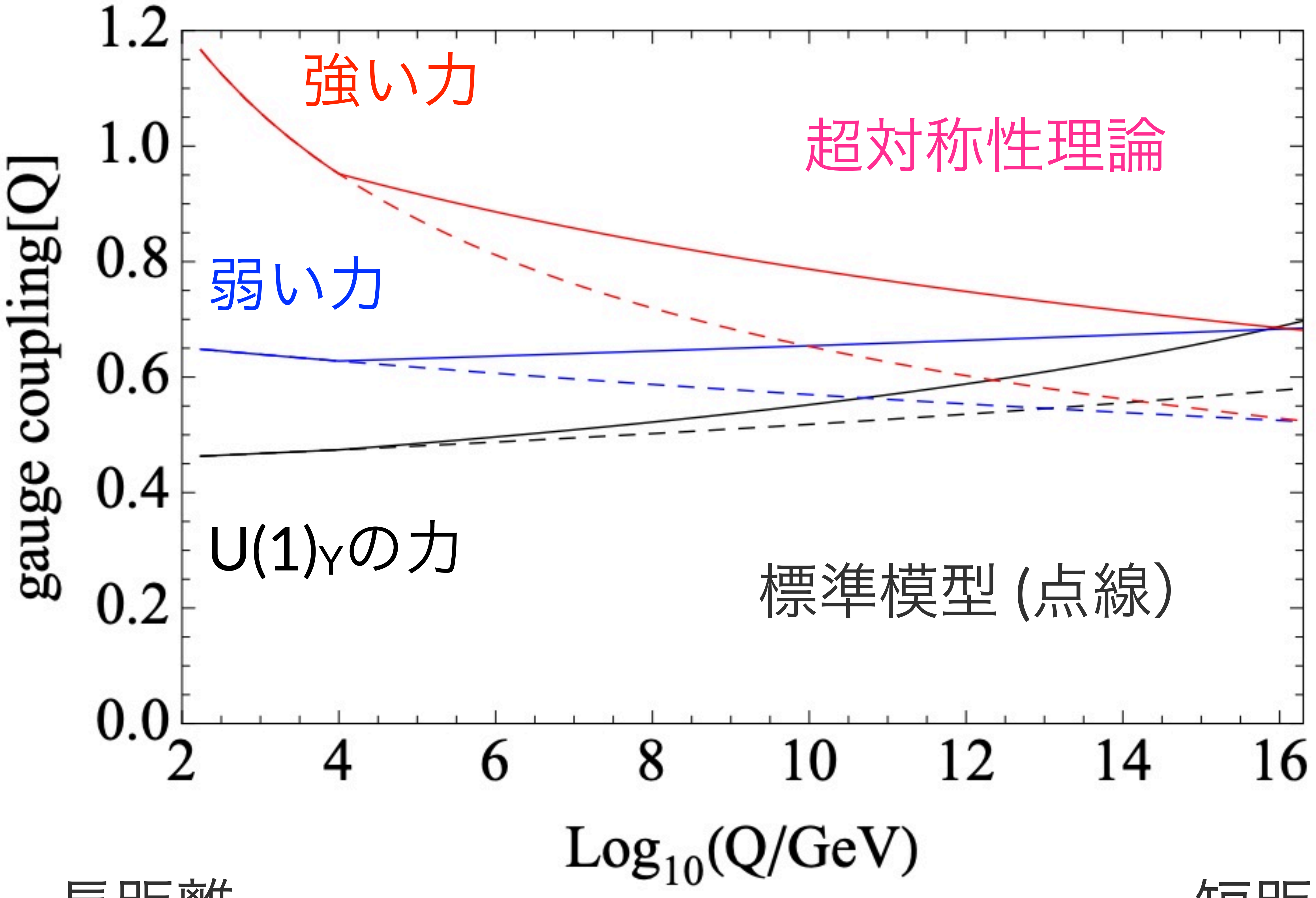
超対称性粒子も18種類

https://ific.uv.es/sct/physics_susy

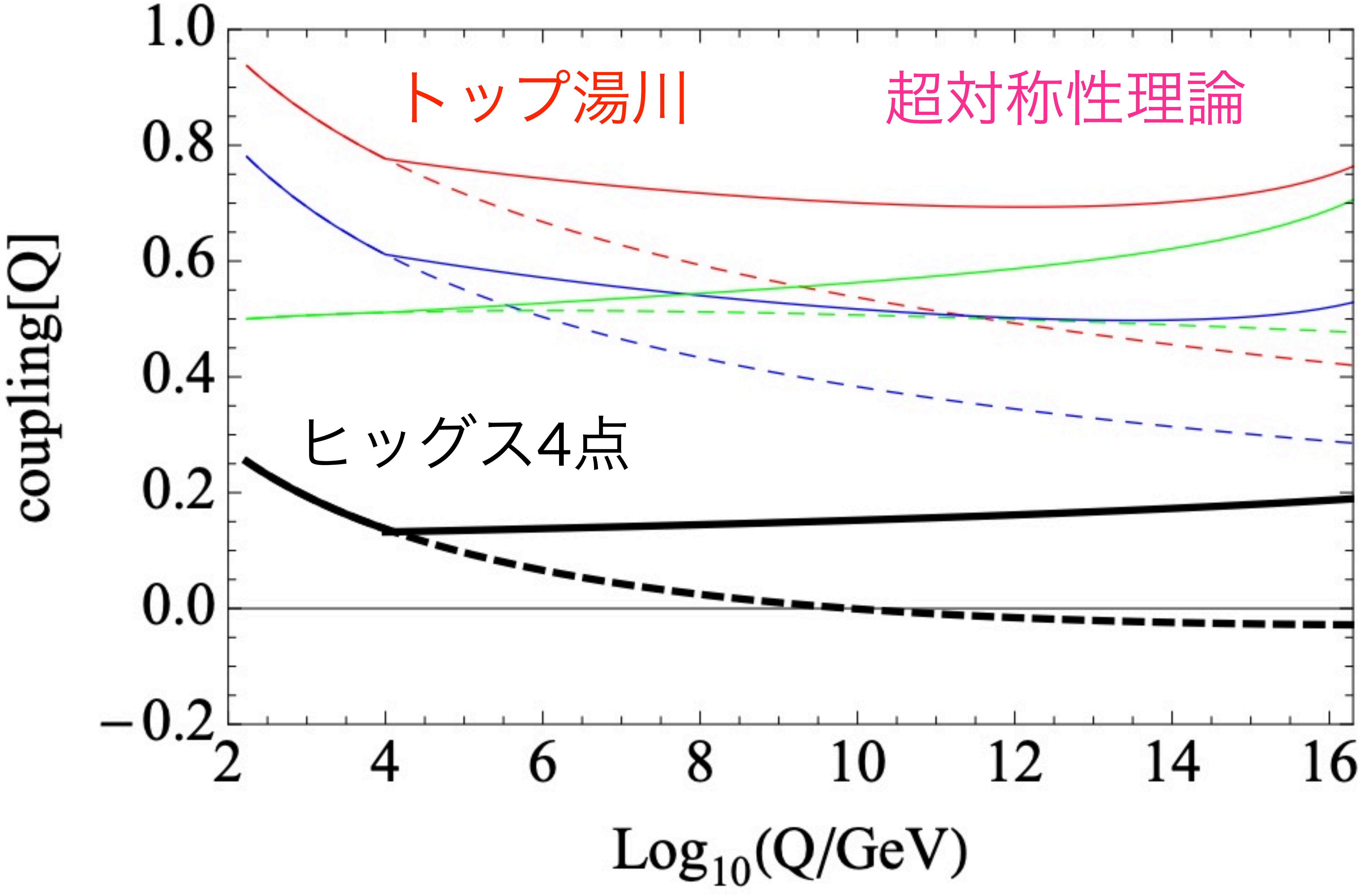
超対称性の五つの裏づけ（？）

- 1. 標準模型の謎の「階層性問題」の解決
- 2. ゲージ場とゲージ対称性の統一、大統一理論の実現
- 3. 最も軽い超対称性粒子が暗黒物質の性質と一致
- 4. 真空の安定性
- 5. 超対称性代数は標準模型の座標変換不変性の自然な拡張として得られた
- 宇宙のバリオン数生成の謎やニュートリノ質量機構との相性も◎

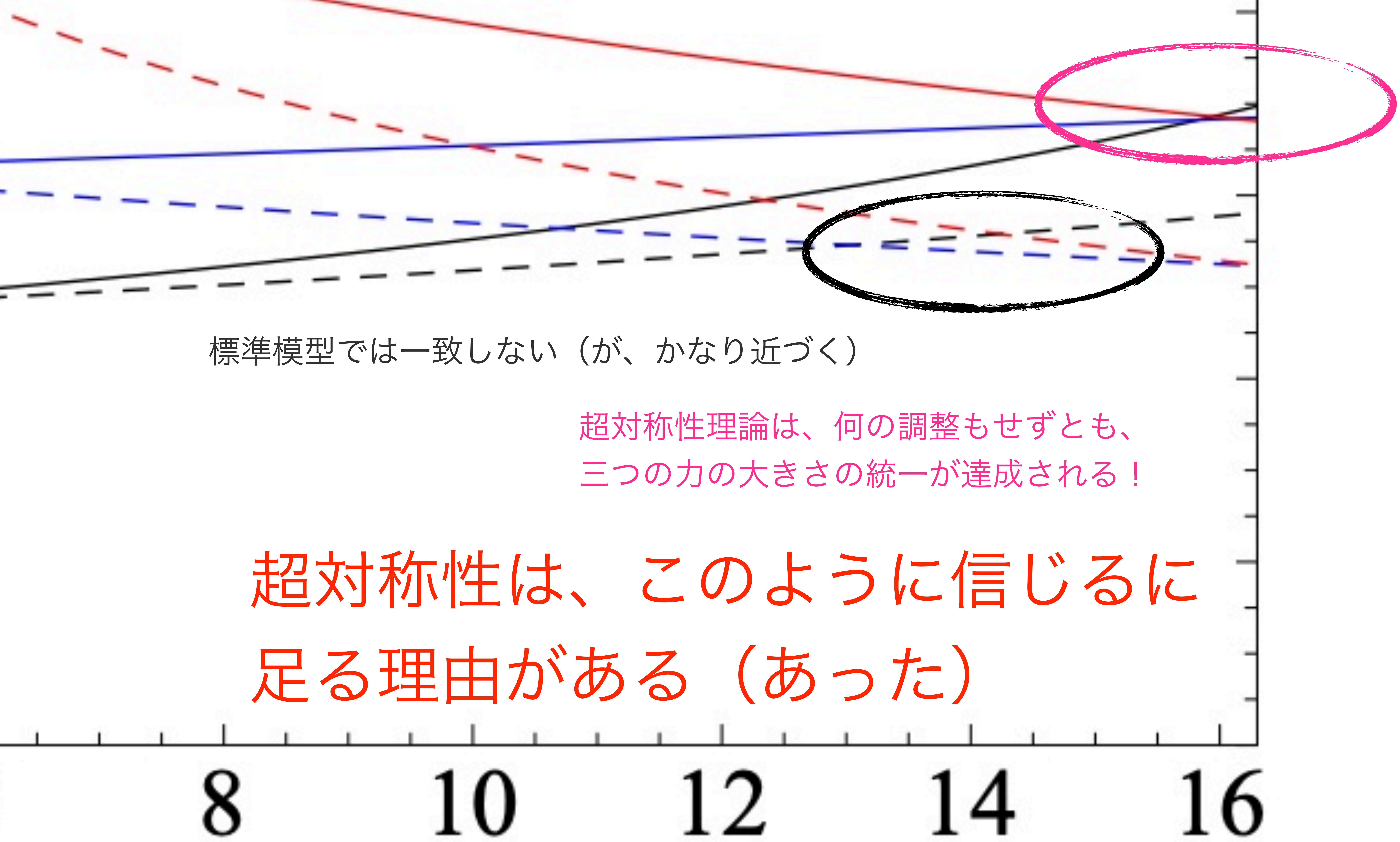
ゲージ対称性の統一 (左) と真空の安定性 (右)



長距離　　短距離



[北原のD論から]



標準模型では一致しない（が、かなり近づく）

超対称性理論は、何の調整もせずとも、
三つの力の大きさの統一が達成される！

超対称性は、このように信じるに
足る理由がある（あった）

超対称性探索: 最新の結果

[ATL-PHYS-PUB-2023-025]